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**A
SCHOOL
ELECTRICITY**

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A SCHOOL ELECTRICITY

BY
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TO
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PREFACE

THIS book is the outcome of notes on a course of lessons at Oundle School. It was by accident that they came into the hands of the University Press for, though I intended to print them for my own use, I had no intention of publishing them.

These lessons were essentially practical: that is to say, there was always laboratory and lecture apparatus at my disposal and all the experiments here described were actually done, some in the laboratory by the boys, some in the classroom. Consequently many important parts of the subject which I did not think suited to class experiment were omitted. No reference therefore will be found in this book to X-rays, telephones and electric waves.

Without an over-large stock of expensive apparatus it is not easy for all boys in a class to do the same experiment at one lesson: nor is it necessary, for most boys now know something in a general way about electricity and in selecting experiments for them to perform I do not feel it necessary to adhere so closely to a logical order as I should were they on entirely new ground. I would suggest however to others who may use this book that at first the lessons should be largely demonstrations: later individual laboratory work may predominate, boys repeating for themselves experiments shewn previously in the theatre by their master.

I believe very strongly in the early use of instruments (e.g. ammeters and voltmeters) which give direct readings in addition to those which only compare (e.g. galvanometers as generally used). It is a great advantage to a beginner to be able to name his current in amperes or his E.M.F. in volts. Of course the complete equipment of a laboratory is expensive, but good instruments which with suitable shunts and resistances cover wide ranges are now so easily obtained and can be used in so many different experiments that I have no

hesitation in urging their adoption. In cases, however, where the cost is really prohibitive, I suggest (Art. 30) that the stock of galvanometers should be calibrated to give direct readings.

I once thought of treating even Frictional Electricity with the help of the Static Voltmeter, etc.: but I was deterred partly by the expense and partly because such treatment diverges too much from ordinary lines.

Ideas of Potential are often found hard; but a short preliminary treatment of Work and Horse-power in the Mechanics lessons takes away most of a boy's difficulties and makes it possible to talk about watts and volts quite early in the study of electricity.

I hope that experiments are described with sufficient detail. It is difficult to strike a mean between over-elaborate directions written round one's own apparatus and generalised instruction so wanting in exactness as to necessitate a special laboratory book.

The examples are mainly original but some have been taken from Scholarship Examinations: others (by permission of the Controller of H.M.'s Stationery Office) from S. K. Papers set in years previous to 1897. Apology might be needed for including some of these but for their annual recurrence slightly altered in other places.

A few, which are much too hard for a junior class and involve the calculus, are inserted for the use of Scholarship Candidates.

I have received valuable help and criticisms from my colleagues Mr E. Jobling and Mr H. Freeman. Figure 116 is reproduced by kind permission of Lady Kelvin and Messrs Macmillan: Figs. 73, 74 come from the catalogue of the Cambridge Scientific Instrument Company. Many of the diagrams of lines of force were drawn by boys. To all these my thanks are due. I am indebted to Mr G. C. Bloomer for the correction of many errata. Finally I wish to express my gratitude to Mr J. B. Peace who has himself read through the whole of the proofs and made many valuable suggestions during the preparation of this work for the press.

C. J. L. W.

July 1914.

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INTRODUCTORY CHAPTER ON MAGNETISM

THE subjects Electricity and Magnetism are so closely related that it will be convenient to devote a few pages to some of the simpler facts of magnetism before proceeding to the study of the phenomena connected with electric currents. Some of the main facts of magnetism are known to almost everyone and may therefore be passed over in very few words.

1. Pole: Axis: Meridian. Take any ordinary magnet and sprinkle it over with iron filings. The filings cling to it: but they do not cling to it evenly: they gather together in two bunches, the centres of which are usually near the ends of the magnet. These centres are spoken of as the *poles*; the line or the direction of the line joining them is the *axis*. In the case of a long thin magnet such as a knitting needle these poles are clearly marked: in a thick horseshoe magnet they are less definite.

If a magnet be suspended in such a way that it can swing freely with its axis horizontal, it will be found that it oscillates to and fro and always comes to rest with its axis in a particular direction. In most parts of the world this direction is nearly North and South, so that we can speak of the two ends or poles of a magnet as the North and the South.

The vertical plane passing through the axis of a freely suspended needle is termed the plane of the magnetic *meridian*.

2. Action of Poles. Bring the North pole of a bar magnet near the North pole of a compass needle: the latter will move away. The result is exactly similar with two South poles. If however the North pole of the magnet be

brought near the South pole of a compass needle the latter will begin to move nearer. Hence it may be said that like poles repel one another, unlike poles attract.

3. Law of the Inverse Square. It is quite readily seen that the nearer two poles are together the greater the attraction between them, but the exact relation between force and distance was not definitely established till the time of Coulomb (1777). Coulomb guessed the law and then proceeded to verify it by experiments on the torsion balance which he had invented. The law is "the force between two magnetic poles varies inversely as the square of the distance between them." Doubling the distance between two poles reduces the force to a quarter of the previous value ; halving the distance quadruples the force.

4. Unit Pole. We require a standard by which to measure pole strength. We first define our unit pole as follows : If two exactly similar poles placed one centimetre apart repel one another with a force of one dyne, each pole is said to be of unit strength.

The strength of any other pole is measured by the force (in dynes) it exerts on a unit pole when the two are placed one centimetre apart. It may further be shewn by experiment that the force exerted by any two poles on one another is directly proportional to the pole strength of each. We have then this law : The force (in dynes) between two poles is measured by the product of the pole strengths divided by the square of the distance (in centimetres) between them.

$$F = \frac{m \times m'}{d^2}.$$

We have here said nothing about the medium in which the poles are placed : strictly speaking they should be *in vacuo* but the results are not appreciably altered if as is usual the medium is air.

5. Strength of Field. The space round a magnet throughout which its influence is appreciable and can be detected by a compass needle or other means is called the field. The strength of a field at any point is measured by the force in dynes that would be exerted on a unit North

pole if it were placed at that point: the unit is the *gauss*, thus at a distance of 5 cm. from a pole of strength 200 the strength of the field is $200 \times 1/5^2$, i.e. 8 gaussess. The strength of the field is also spoken of as the *intensity of field* or the *magnetic force*.

6. The Earth's Magnetic Field. The earth has a magnetic field, the intensity and direction of which are different in different places. In England the field is northerly and inclined to the horizontal at an angle of about 67 degrees. An ordinary compass, by the direction in which it points, shews that this field is northerly: it fails to shew that the field is not horizontal because it is so mounted that it can only swing round in a horizontal plane. If it were free to tilt it would point with its North pole downwards at about 67 degrees.

Consider now a North pole of unit strength. In England this will be pulled northerly and downwards by a force of about half a dyne. Resolve this force into two components; the one vertically downwards, the other horizontal and North. The latter is called the horizontal component of the Earth's magnetic field: it is usually denoted by the symbol H : its value in London is about .186 dyne. It is this horizontal component that regulates the behaviour of a compass needle: the vertical component has no effect upon it.

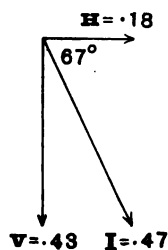


Fig. 1.

7. Lines of Force. If a bar magnet be laid on a table and be covered with a card on which iron filings are sprinkled, a few gentle taps will cause the filings to range themselves in chains joining the two poles together: see the plates at the end of the book. These chains mark out what are called lines of force. We may define a line of force as the course that would be traced out by a North pole, free to move, under the influence of the magnetic force. The tangent to such a line at any point is in the direction of the resultant magnetic force.

8. Resultant Field. The magnetic force at a point is often the resultant of two or more components. The case is

important in which we have to find the direction of the resultant of (1) the Earth's horizontal component, $\mathbf{H}$, (2) a horizontal force $\mathbf{F}$ acting East or West. To do this draw from a point O a line OA to represent $\mathbf{H}$ in magnitude ; and

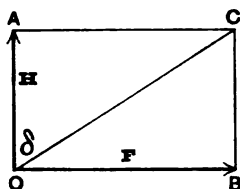


Fig. 2.

another OB to represent $\mathbf{F}$. OB will of course be at right angles to OA . Complete the parallelogram $OACB$. Then the line OC gives the direction of the resultant magnetic force. A compass needle placed at O would point along OC . It would be deflected from the North through the angle AOC .

$$\tan AOC = \frac{AC}{OA} = \frac{OB}{OA} = \frac{\mathbf{F}}{\mathbf{H}}.$$

The angle AOC is frequently denoted by δ ; hence

$$\mathbf{F} = \mathbf{H} \tan \delta.$$

CHAPTER I

ACTION OF CURRENT AND POLE

9. Cells and Batteries. In these days everybody is familiar with some of the uses of electricity. In all large towns lamps are lighted by electricity, workshops and trams are driven by it. The source of supply in such cases is usually a dynamo driven by steam or water. In telegraphy, telephony and for ringing bells, cells, or batteries of cells, are generally employed. In a physical laboratory it is probable that there is an electric light installation in connection with the town mains and this will often be a convenience. For many experiments however we still prefer to use cells of some kind.

Every cell has two poles, generally of different materials. In a Daniell they are copper and zinc, in a Leclanché they are carbon and zinc; in a storage cell (or accumulator) we have two lead grids containing pastes of different materials. In any cell one of the poles is called positive (+), the other negative (-). In a storage cell the negative pole is often painted black, the positive red: the positive plate contains a chocolate coloured paste.

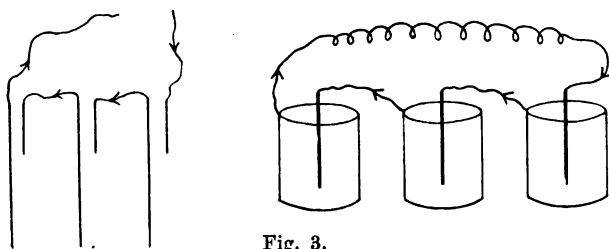


Fig. 3.

Several cells may be grouped together to form a battery. If the cells are arranged with the + of one joined to the - of the next, as in fig. 3, they are said to be in series. This

is a usual method if we want to drive a big current through a large resistance.

If the cells are joined up by connecting up all the positive poles together, and all the negative together, they are said to be in parallel, or abreast, or in multiple arc. Fig. 4 is a diagram of a battery of three cells abreast. We shall discuss later other ways of joining up batteries.

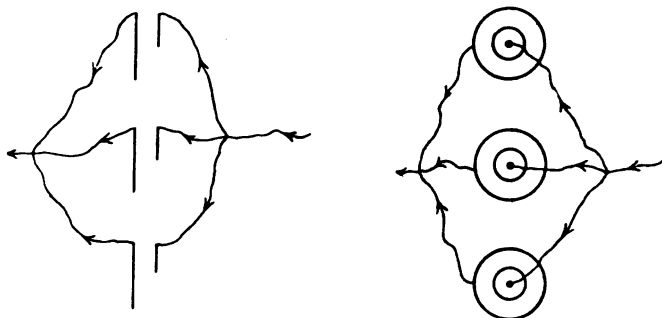


Fig. 4.

The names positive and negative have been given quite arbitrarily; it would have done just as well if the zinc had been regarded as positive and the copper or carbon as negative.

If the poles of a cell are joined by wire, some things happen in the wire itself, other things happen in the neighbourhood of the wire outside it: and the wire is said to carry a current of electricity. We do not know what a current of electricity is, and we do not know in which direction it goes, but for convenience we always talk of it as if electricity passed along the wire from the positive pole to the negative outside the cell: and as electricity always flows in a complete unbroken circuit, it follows that we must regard it as passing from the negative to the positive inside the cell itself (fig. 5).

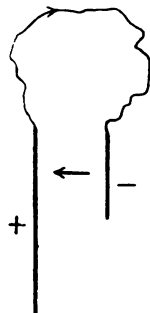


Fig. 5.

10. Oersted's Experiment. We owe to Oersted the discovery of the magnetic effects of a current of electricity. In 1819 he placed a wire

joining the two poles of a cell over a compass needle. He found that the needle was deflected. To repeat his experiment place a compass on the table, stretch a wire over the needle and parallel to it, join the South end of the wire to the positive pole of a cell, the North end to the negative pole. The current now passes over the compass from South to North. The needle is deflected and you will see that the North pole points somewhere between North and West.

Repeat the experiment with the current reversed; also with the wire below the compass needle and the current in each direction. Double the wire on itself and find what effect the current in the doubled wire has on the needle.

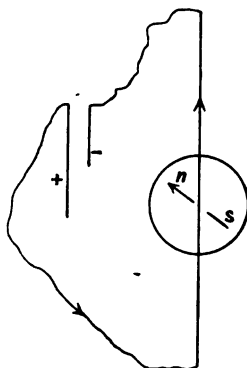


Fig. 6.

Make a coil of three or four turns of wire: hold it in the meridian and place a compass needle in the centre. Join up the ends of the wire to the poles of the cell and note the effect on the needle. Such a coil of wire with a compass at the centre makes a simple galvanometer: it detects currents and shews their direction.

The results of these experiments give the following simple rule for finding in which direction a compass needle tends to point when under the influence of a current. Imagine that you are swimming parallel to the current and facing the conductor: if you stretch out your arms at right angles to your body, the right arm points in the direction in which the North pole of a compass (situated between yourself and the conductor) tends to point.

If a current flows in a straight conductor, a compass needle in its neighbourhood would always be at right angles to it if it were unacted on by any other force. This is not often the case for there is nearly always a magnetic field due to the earth. Thus in the first part of the experiment described above, the current tends to make the needle point West, the earth tends to make it point North. The direction in which it actually does point is one between North and West: it is the direction of the resultant of the two forces.

11. Lines of Force. The nature of the field round an ordinary bar magnet can be investigated by sprinkling iron filings on a sheet of cardboard: we may use the same method for surveying the magnetic field round a current. Take a simple case. Arrange a stout vertical wire to pass through the middle of a horizontal sheet of cardboard. Pass a strong current through the vertical wire: this may conveniently be done by joining up a few cells in parallel. Sprinkle iron filings on the cardboard and tap gently. The filings will arrange themselves in circular rings round the wire (fig. 7).

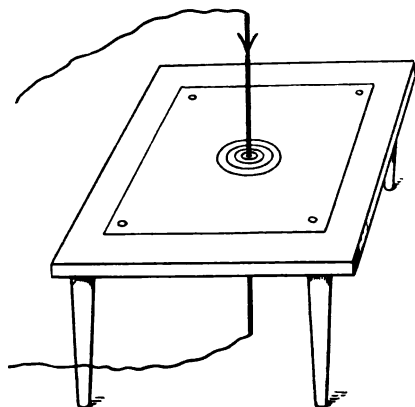


Fig. 7.

The lines of magnetic force then due to a current in a straight conductor are circles with their centres in the conductor. Now lines of force are the paths that the North pole of a compass would trace if allowed to move slowly from point to point. Hence if we could imagine we had a North pole all by itself, it would travel round and round a straight conductor in a circle. The direction in which it would trace the circle is shewn by fig. 8. We call this the right hand screw direction: it is the direction in which a corkscrew rotates when it is being screwed forward in the direction of the current. The South pole of a compass needle tends to travel in exactly the reverse way. Suppose we think about the forces on the two poles of a compass needle due to a current flowing due North in a horizontal conductor held directly above. The corkscrew rule tells us that the force on the North pole will

be due West, that on the South pole due East. Hence the compass has a tendency to point at right angles to the earth's magnetic meridian, the North pole towards the West. This result has already been obtained.

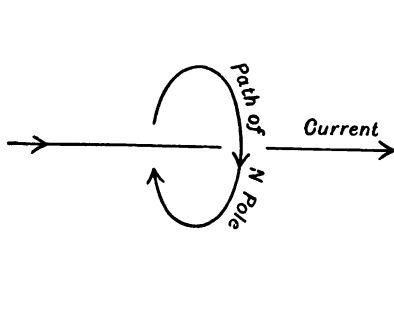


Fig. 8.

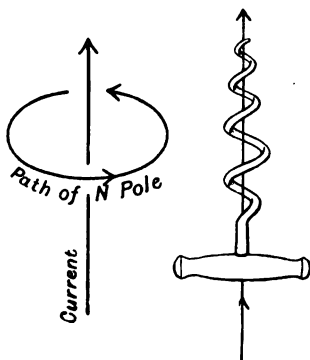


Fig. 9.

The lines of force considered so far have been supposed to be due to the current only. If we add on the earth's field of force the lines will no longer be circular. The combination of the two gives us fig. 10. Here the shaded part is supposed to be a section of the conductor. The + mark on the section of a conductor indicates that the current is going down through the paper: think of it as the feathered tail of a retreating arrow. To indicate a current coming up through the paper we mark the section with a •, the point of an advancing arrow.

This figure cannot be obtained by means of iron filings: the method is not delicate enough, except for places quite close to the conductor. A convenient way to trace such a figure is by means of a small compass needle. Lay this anywhere on the card and make a dot under each pole. Now move the compass to such a position that the South pole is where the North pole was: put another dot under the North pole in its new position and move the compass on again. You get in this way a series of dots which are to be joined up to give a line of force. Trace several others by selecting new starting points. There are other cases in which a knowledge of the lines of force in a field is of great importance. Some of these will be discussed later; the cases

already described have been selected to render you familiar with the idea that there is a sort of magnetic whirl round every current.

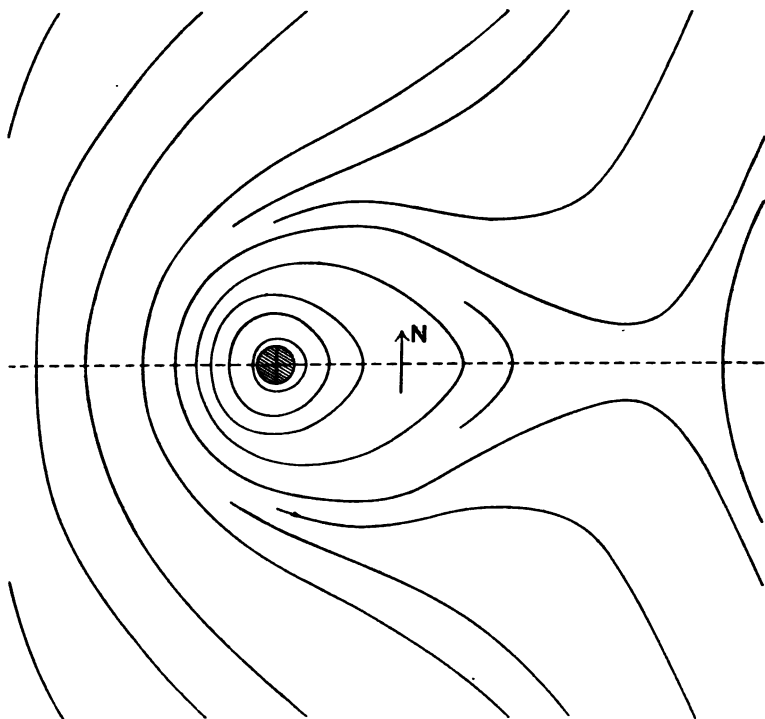


Fig. 10.

12. Rotation of magnets. If we could get an isolated pole we should expect it to circle round and round the current in a conductor : in this way we should get a simple motor. It is however impossible to get one pole all by itself so we have to adopt some device to neutralise the effect of the other if we wish to produce rotation.

A simple arrangement is shewn in fig. 11. Here two magnets, *SN*, *SN*, are hung by a fibre *F* attached to a hook *H*. They are connected together by the conducting bar *BB* at the ends of which are spikes dipping into the circular trough (*AA*) of mercury. The cup *C* also contains mercury and is connected to the bar *BB* by the straight wire *D*.

Now if a current is passed into C up D , through the bar BB into the channel AA , the suspended magnets will rotate round the vertical axis.

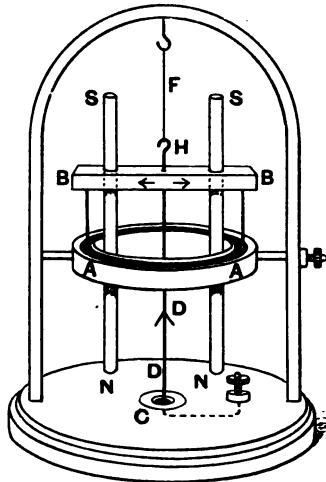


Fig. 11.

13. Action on Conductors. A current exerts a force on a magnetic pole: to this action we expect some sort of reaction: we expect that a magnetic pole exerts a force on a conductor carrying a current. Suppose AB (fig. 12) is a vertical wire in which a current is flowing upwards, and that NS is a horizontal bar magnet. Now if the wire is fixed and the magnet free to move, as it is in the case of a compass, the corkscrew rule tells us that the North pole N will begin to move away from us, down into the paper. If however the magnet is fixed and the wire free, the wire will begin to come towards us under a force exactly equal in magnitude but opposite in direction to that exerted by the current on the pole. If a second magnet ns is placed as in the figure the effect on the wire will be increased. The experiment may easily be performed by passing a current through a flexible wire suspended between the poles of an electromagnet.

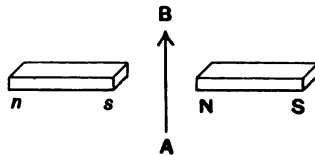


Fig. 12.

14. Barlow's Wheel (fig. 13) is a simple application of this. The rim of the wheel *W* grazes the surface of the mercury *M* in a little cup. The wheel can rotate on a horizontal axis *AA* supported on bearings (not shewn). On opposite sides of the lower part of the wheel are placed unlike magnetic poles *N, S*; a current led up from the mercury between the poles *N, S* and taken off from the axis causes the wheel to turn. The wheel is generally made stellate but this form is not necessary.

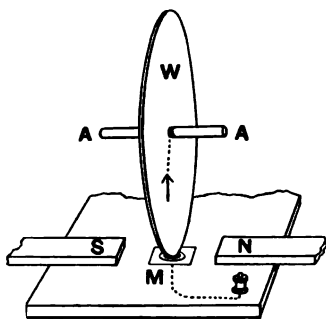


Fig. 13.

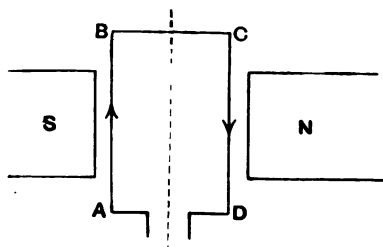


Fig. 14.

15. A simple motor. *ABCD* (fig. 14) is a bent wire placed between the poles *N, S*. By the reasoning given above we see that the current flowing along *AB* tends to make *AB* move upwards out of the paper. The current in *CD* is in the opposite direction, so that *CD* tends to move down into the paper. Hence if the wire is free to move it will begin to rotate round the dotted line. The next diagram

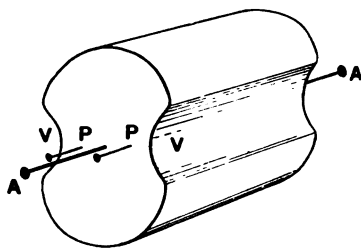


Fig. 15.

(fig. 15) shews how this may be done quite simply. Pins *AA* are driven into the centres of the faces of a bottle cork.

These project $\frac{1}{2}$ inch and form an axle. *V*-shaped grooves *VV* are cut in the surface, on opposite sides, parallel to the axis. Into one end two pins *PP* are driven projecting $\frac{1}{4}$ inch. Insulated wire (about No. 22) is now wrapped round and round the cork, lengthwise, till the grooves *VV* are filled up: the bared ends of this wire are connected to the pins *PP*. The whole may now be supported as in fig. 16 with the axle

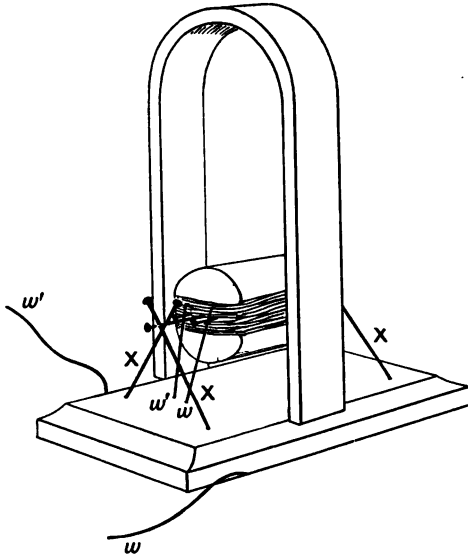


Fig. 16.

horizontal, on four large pins, two at each end crossed at right angles (*X*, *X*). The pins *AA* rest in the angles formed at the intersection.

A current is led up a thin springy piece of copper wire *w* which rests lightly against *P*; it then passes round and round the cork; and finally leaves by a second wire *w'*. The cork is placed between the poles of a large horseshoe magnet. The current from a single cell will run this simple motor.

16. Luminous discharge. If a current of electricity is passed through a vacuum tube by means of an induction

coil, the gas will glow. The tube may be so constructed that the pole of a magnet can be inserted into it. The rotation of the luminous conducting gas can be seen in this way.

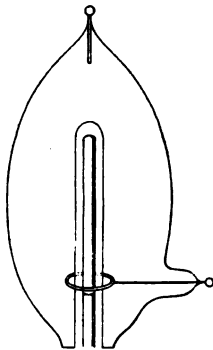


Fig. 17.

EXAMPLES I

1. Draw a diagram to shew two cells joined in parallel arranged to ring a bell.
2. Draw a diagram of four cells joined in series arranged to light a lamp.
3. Describe with diagrams the effects of currents in straight horizontal wires on a compass needle.
4. In what ways is it possible to find in which direction the current in a wire is flowing?
5. A coil of wire is laid on the table and a current passes round it clockwise. Compass needles are placed over the wire at different points. Shew by a diagram the directions in which they will point.
6. A loose flexible wire hangs vertically between the poles of a strong horseshoe magnet, the plane of which is horizontal. How will the wire be affected if a current is passed down it?
7. A current flows down a vertical wire held a little due East of a compass needle. Shew the direction of the forces acting on each pole. Will the compass needle move?

8. A loose flexible wire hangs vertically in the earth's field. Is there any tendency for it to move when a current passes down it?

9. A bar magnet lies on the scale-pan of a balance. Underneath the pan, at right angles to the magnet, is a wire which carries a current. Shew by a diagram the action of the current on each pole. Will the current alter the apparent weight of the magnet?

10. The keeper of a horseshoe magnet is hinged freely to the magnet at one pole. Through the bend of the horseshoe at right angles to the axis passes a wire carrying a current. Can this current produce any effect on the keeper? If so, suggest an arrangement by which a current might be measured.

CHAPTER II

ELECTROLYSIS

17. In the last chapter we considered some of the effects that were produced in the space surrounding a conductor conveying a current. We are now going to study the action of a current in the conductor itself. If the conductor is a metal the action is not very marked: the conductor gets warmed but its state does not seem to be altered in other respects. In the case of a liquid conductor the current generally produces chemical change. The only liquids we shall deal with are solutions in water. Water itself and nearly all liquids except solutions (e.g. oil, alcohol, turpentine, sulphuric acid) seem to have little or no power of conducting electricity.

To study the action of a current on a solution we must have two conductors dipping into the solution: these are called *electrodes*. The electrode by which the current enters

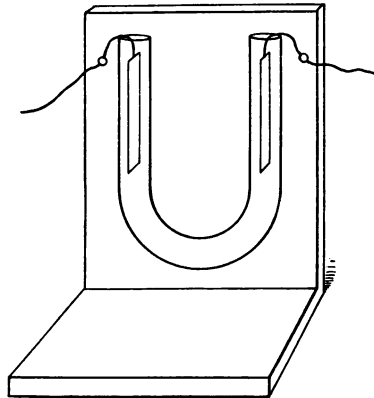


Fig. 18.

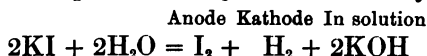
is called the *anode*, that by which it leaves is called the *kathode*. The electrodes are often of platinum but in some cases may be of other metals or of carbon. The vessel in which the solution is held is called a *voltameter*. A convenient form of voltameter for some experiments is shewn in fig. 18. It consists of a glass U tube to hold the solution. In each arm is a leaf of platinum connected to a wire.

18. Electrolysis of Potassium Iodide. Take a voltameter with platinum electrodes and put in it a dilute solution of potassium iodide. Connect up to a battery of two or three cells. Notice carefully the action that takes place at each electrode. After the current has been stopped, look at the platinum and see if they have been affected in any way.

The dark deposit that formed round the anode was iodine: it can easily be tested by starch coloration. The gas that bubbled off from the kathode was hydrogen. If sufficient can be collected it should be tested. The action that has taken place appears to be

(1) the removal of the iodine from the potassium: the iodine appears at the anode;

(2) the potassium deprived of its partner unites with the water to form potassium hydroxide and hydrogen.



The decomposition of a solution by a current of electricity is called *electrolysis*.

19. Electrolysis of Sodium Sulphate. Use a Hoffmann's voltameter with platinum terminals, fig. 19. Colour the solution with a little litmus. Notice the action at each electrode. Collect the gases and test them. After the action has been going on for a little time, allow the gases to escape and reverse the current. Notice how the coloration changes. At the end of the experiment mix the liquids round the electrodes together.

The reactions in this experiment are exactly similar to those in the electrolysis of potassium iodide. The sodium sulphate (Na_2SO_4) seems to be separated out into two portions; sodium which we know and can isolate, and the

sulphate group (SO_4) known as sulphion which we cannot isolate. The sodium is of course not given up in the metallic form at the kathode. It acts on the water, hydrogen is liberated and sodium hydroxide is formed round the kathode: it is this sodium hydroxide which gives the blue coloration to the litmus. There is also a chemical action round the anode: the sulphion and water together give rise to oxygen and sulphuric acid which turns the litmus red; the oxygen bubbles off from the platinum anode.

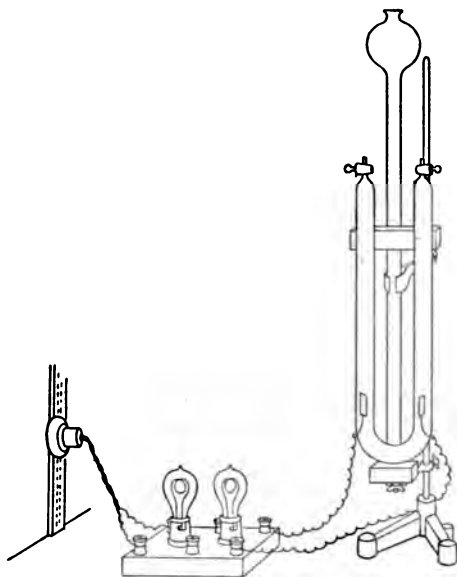
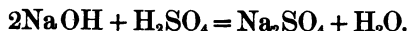
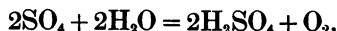
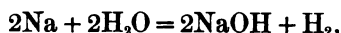
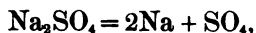


Fig. 19.

When a substance is split up by a current, the components are called *ions*. That which appears at the anode is called the *anion*; that at the kathode is the *kation*. In the last experiment the ions were sulphion and sodium; the sulphion was the anion, the sodium the kation. The original solution is termed the *electrolyte*.

When the two portions of the liquid are mixed together the sodium hydroxide acts on the sulphuric acid and so returns to us sodium sulphate and water. The total result is that some of the water has been decomposed into hydrogen

and oxygen. There is exactly the same quantity of the sodium sulphate as before. What we have effected is the electrolysis of water: this electrolysis however cannot take place unless there is some substance dissolved in the water.



20. Pole-Testing Paper. Take a dilute solution of sodium sulphate and add to it a little phenolphthalein. Use the mixture to moisten a piece of absorbent paper. Now touch one part of the paper with a wire from the positive pole of a cell and another part with a wire from the negative. The spot touched with the negative wire will be turned a bright red. The reason of this is that sodium hydroxide is formed round the kathode. Any alkali turns phenolphthalein red.

21. Electrolysis of dilute sulphuric acid. Do this with a graduated Hoffmann's voltameter. Notice the gases at each electrode and test them. Note the volumes.

22. Electrolysis of copper sulphate. What would you expect if platinum electrodes are used? Do the experiment to test your answer.

Next set up a copper voltameter. To do this take two plates of copper with terminals attached to them and let them stand in a vessel containing a solution of copper sulphate. They must be clean before the experiment begins. The current must pass for some time before you can see what has happened.

In the electrolysis of copper sulphate, we have two ions; copper and sulphion. Whatever the nature of the electrodes is, copper will be deposited on the kathode. If the anode is platinum the sulphion in conjunction with the water produces oxygen and sulphuric acid. The copper of the copper sulphate is therefore gradually removed from the solution and deposited on the kathode. The solution becomes paler as the copper sulphate is gradually replaced by sulphuric acid. If however the anode is of copper, the sulphion unites with it and forms copper sulphate. Hence as a result of the passage of the

current we get copper dissolved off the anode and exactly the same amount deposited on the kathode. If the anode were of copper and the kathode of platinum or carbon, the current would convey copper from one to the other so that the kathode would be copper plated. Silver plating is done by using a silver anode and a solution of silver salt (nitrate or cyanide) as electrolyte: the article to be plated is the kathode.

23. Change in concentration. In the electrolysis of copper sulphate and of other electrolytes certain changes in the solution take place which are not readily observed in the ordinary voltameter. To shew this take a long wide glass tube; fit it with a cork at each end. Pierce the corks and pass through them thick copper wires to serve as electrodes. The tube must be held vertical in a clamp and filled to such a depth with copper sulphate solution that the upper wire dips right into it. Now pass a current up the tube so that the lower end is the anode. You will find that changes take place in the colour and density of the solution. Of course on the whole there is no change in the total quantity of the copper sulphate solution, but as the current passes the concentration becomes greater near the anode but less near the kathode to a corresponding extent. Liquid diffusion prevents all the salt from accumulating round the anode and leaving the kathode clear.

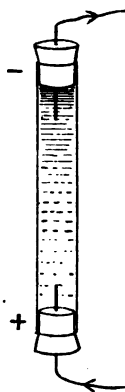


Fig. 20.

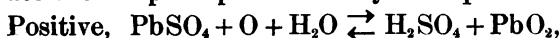
24. Secondary cells. Take a pair of lead plates, say 8 inches by 2 inches, and use them as electrodes in an electrolytic cell. The electrolyte is to be dilute sulphuric acid. Pass a current between them for a few minutes, then disconnect the battery and join up the terminals of the lead plates to a galvanometer. What is the direction of the current now given? Exactly the same effect is produced from the platinum in a Hoffmann's voltameter after it has been used for electrolysis of water in the ordinary way.

In 1859 Gustav Planté experimented with lead electrolytic cells, trying to make an accumulator or storage battery. By frequent charging—first in one direction and then in the reverse—he succeeded in “forming” the plates: that is to say in bringing the surface to a softer condition and so enabling them to take a much larger charge.

The modern accumulator is not made quite in this way. In some kinds the electrodes are lead grids; the positive filled with a paste of red lead (Pb_3O_4), the negative with a paste of litharge (PbO). The electrolyte is dilute sulphuric acid. The acid acts on the red lead to form the peroxide (PbO_2) and the sulphate (PbSO_4).

In the process of charging the positive plate, which is of course the anode, is further oxidised while the negative is reduced. In the discharge the action is reversed. Needless to say there is no actual storage of electricity: it is chemical energy that is acquired by the charge and expended by the discharge.

The actions are perhaps denoted by the equations



The formation of sulphuric acid during the charge accounts for the increase of density of the electrolyte which may be observed.

25. The chemical equivalent of copper. It is now necessary to do some quantitative work in electrolysis. We will begin by passing a current through two voltmeters in series. The first has platinum electrodes in dilute sulphuric acid, the second has copper electrodes in a solution of copper sulphate. Since the voltmeters are in series, exactly the same current passes through each for the same time. The copper kathode must be weighed carefully before and after the current is passed. The volume of hydrogen liberated at the kathode must be found. We can then calculate the weight, for the density of hydrogen is .09 grm. per litre. Correction should of course be made for temperature, pressure and the presence of aqueous vapour. The ratio of the weight of the copper deposited to the weight of the hydrogen evolved gives us what is known as the chemical

equivalent of copper. Calculate the value of this from your results. To get good results with a copper voltameter attention must be paid to (1) the strength of current, (2) concentration of the solution, (3) cleaning and drying of the plate.

(1) A suitable strength is 1 ampere for every 50 sq. cm. of surface on which copper is to be deposited. If the current is much stronger than this, the deposit is not coherent.

(2) Make the solution by dissolving 300 grms. of copper sulphate in a litre of distilled water and adding 10 c.c. of strong sulphuric acid.

(3) Rub the plate with a rag and damp sand till the surface is quite bright. Rinse under a tap. Dry it by first laying it between filter papers and then warming it gently near a fire.

26. The Ampere. It is usual to define the strength of a steady current by the quantity of a metal deposited per second by it. The metal selected is silver which is readily deposited in a coherent form and does not oxidise quickly. *The ampere is that current which deposits silver at the rate of '001118 gm. per second.*

Now silver is too expensive to work with in a school laboratory so that for our purposes we shall use copper as the standard. The quantity of copper however will not be '001118 gm. but '00033 gm. An ampere then is a current which deposits copper at the rate of '00033 gm. per second.

27. To test the readings of an ammeter, or to standardise a galvanometer. We shall do this by passing a current through a copper voltameter. Take a copper voltameter; clean the plates. Now join up in series with

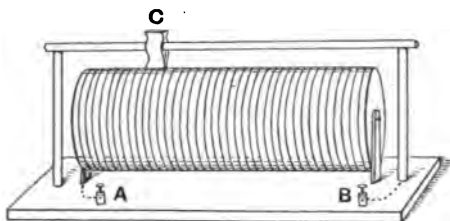


Fig. 21.

a constant battery (1) the voltameter, (2) the ammeter or galvanometer, (3) a variable resistance or rheostat. A convenient form of rheostat is shewn in the diagram, fig. 21. It consists of wire spiral wound on a cylinder of slate. One end is connected to the terminal *A*. A sliding contact *C* is joined to the terminal *B*. The further *C* is removed from *A* the greater the effect in reducing the current. Adjust the rheostat till the ammeter gives a reading on that part of the scale at which it is required to make a test. Let the current pass for a few minutes then stop it. Take out the copper plate which has been the kathode, wash and dry it and weigh carefully. Replace it and start the current again. Make a note of the exact time at which you do this. The reading of the meter must be kept constant during the whole of the experiment. You can do this by regulating the rheostat. The time required depends on the accuracy with which you can weigh and the strength of the current; thus if the current is 3 amperes, you will get an increase of rather more than a gramme in 20 minutes. After sufficient copper has been deposited stop the current and note the time. Remove the kathode plate, wash, dry and weigh. Divide the increase in weight in grammes by the time in seconds. Divide this again by '00033. This gives the number of amperes in the current.

28. Faraday's Laws. It might be thought that the rate at which an ion was deposited by a current was not directly proportional to the strength of the current. Either the nature or the size of the electrodes, the strength of the solution or its temperature, might make a difference. About 1833 Faraday made many experiments to find this out. The results of his work are summed up in the statements known as Faraday's Laws of Electrolysis.

(1) The mass of an electrolyte decomposed is proportional to the quantity of electricity which passes.

(2) The mass of an ion liberated by the passage of a definite quantity of electricity is proportional to the chemical equivalent of the ion.

We may gather the two laws together in the statement that the mass of an ion liberated per second is to be found by

multiplying the product of the strength of the current and the chemical equivalent of the ion by the factor '0000104.

These laws may need a little explanation. Refer back to the experiment in Art. 25. Here we found that the same current deposited 31·5 grms. of copper for every gramme of hydrogen and for every gramme of hydrogen there are 8 grms. of oxygen. These numbers, 31·5, 8, are the chemical equivalents of copper and oxygen and may be obtained without the aid of electricity at all. Now an ampere deposits '00033 grm. of copper per second, or it can liberate $'00033 \div 31\cdot5$, i.e. '0000104 grm., of hydrogen per second, or '000083 (i.e. $8 \times '0000104$) grm. of oxygen per sec. The numbers '0000104, '00033, '000083 are termed the electro-chemical equivalents of hydrogen, copper and oxygen. They give the number of grammes liberated per second by a current of one ampere: they may be obtained by multiplying the chemical equivalent by '0000104.

29. The coulomb is a quantity of electricity. A current of a coulomb per second is an ampere. We may say then that a coulomb is a quantity of electricity equal to an ampere-second or that an ampere is a current of a coulomb per second. For every coulomb that passes through an electrolytic cell the electro-chemical equivalent of each ion is liberated. Thus a coulomb will deposit '00033 grm. of copper, or '001118 grm. of silver, or will liberate '0000104 grm. of hydrogen.

30. Tangent Galvanometers. It will be easily understood that a meter after the fashion of a voltmeter might be constructed to measure quantities of electricity. Some forms are on the market, but they do not seem to be very common. Instruments which give direct readings of currents in amperes are called ammeters. In laboratories however direct reading instruments are not always required and consequently tangent galvanometers are often used. The principle and construction of these will be described later. Meanwhile it will be sufficient to say that the usual laboratory form consists of a compass needle mounted at the centre of a ring on which several turns of wire are wound. The plane of this must be in the meridian. The compass needle will point North unless a current flows round the ring; the effect of the current is to

deflect the magnet and this deflection is measured on a graduated card by means of a long pointer attached to the needle at right angles. When the compass is North and South this pointer will be East and West over the zero graduation.

The deflection is dependent on (1) the number of turns of wire, (2) the diameter of the ring, (3) the strength of current, (4) strength of the Earth's field.

When working with them be careful that

- (1) the base is level,
- (2) when no current is passing, the plane of the coil contains the axis of the compass needle—i.e. it must be in the meridian. The pointer should then be at zero.
- (3) no iron is in the neighbourhood.

Galvanometers have often several windings: for instance by connecting up to different terminals you may be able to use 5 turns, or 50, or 500. The actual number ought to be marked on each instrument. For measuring big currents use only a few turns: for small currents use a large number. The tangent galvanometer is so called because the tangent of the deflection is proportional to the current: hence the amperage of a current producing any given deflection may be found by multiplying the tangent of the deflection by a certain quantity: this multiplier is called the *reduction factor*. Thus, if C denote a current measured in amperes which gives a deflection δ , then $C = k \tan \delta$, where k is the reduction factor. To find k and so standardise a galvanometer it is sufficient to pass a known current through the instrument and note the deflection. Thus suppose a current of 2.83 amperes gives a deflection of 29° . Then

$$2.83 = k \tan 29^\circ = k \times .554,$$

$$\therefore k = 5.1,$$

and the current corresponding to a deflection 53°

$$= k \tan 53^\circ$$

$$= 5.1 \times 1.33$$

$$= 6.78 \text{ amperes.}$$

When the reduction factor on a galvanometer is once found with accuracy it will be useful to plot a curve connecting

deflection with amperage. Such a curve is shewn in fig. 22 for the particular example given: the values have been calculated from tables of tangents. Or a simple graphic method would do. Draw a right-angled triangle AOD (fig. 23) on any convenient scale such that the angle AOD is equal to the deflection corresponding to a known current in amperes. In this particular case we might draw AD of length 2.83 cm. to correspond to the current of 2.83 amperes which produces a deflection of 29° . Draw the quadrant AB and mark off along it the degrees.

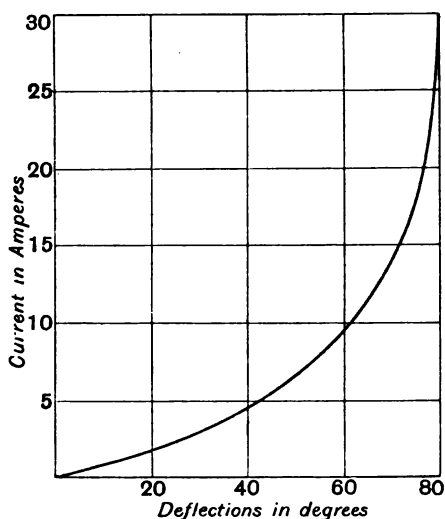


Fig. 22.

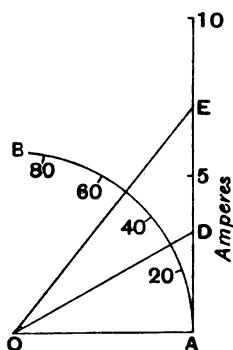


Fig. 23.

Then the current which corresponds to any other deflection (say 53°) can be found by measuring the intercept EA made on AD by the radius inclined at 53° to OA .

No reliance can be placed on values calculated from deflections in the neighbourhood of 0° or 90° ; for very small mistakes in reading such angles give rise to serious errors. Thus $\tan 82^\circ$ differs from $\tan 83^\circ$ by 14 %, and $\tan 3^\circ$ differs from $\tan 4^\circ$ by about 34 %, but $\tan 45^\circ$ differs from $\tan 46^\circ$ by only $3\frac{1}{2}\%$.

31. To standardise a galvanometer to measure amperes. Find the reduction factor in one of the following ways :

(a) by deposition of copper, see Art. 27,

(b) by calculation from the formula of Art. 90,

(c) by an accurate ammeter. Take a cell or battery of cells, a rheostat (or set of resistances), and join up in series with an ammeter and galvanometer. Find the current which produces a deflection of about 45° . Then as in Art. 30 draw up a graph to connect deflection and current.

EXAMPLES* II

1. What actions take place when a current is passed through a voltmeter with silver electrodes filled with a solution of silver nitrate?

2. A current of .75 ampere flows between two plates of copper immersed in a solution of copper sulphate. What changes take place in five minutes?

3. Calculate the number of ampere-hours required to obtain 5 kg. of oxygen by electrolysis.

4. When Davy first obtained potassium he was not absolutely certain that it came from the caustic potash: it might have come from the electrodes or even from the air. Can you suggest any experiments he made—or might have made—to test this point?

5. In the electrolysis of water it is found that the volume of hydrogen obtained is not exactly double that of oxygen. What causes could account for this?

6. During an experiment made with a copper voltmeter to find whether an ammeter was correctly graduated or not, the following readings were taken at the end of successive minutes :—2.50, 2.45, 2.42, 2.41, 2.41, 2.41, 2.40, 2.40, 2.40, 2.40, 2.39.

The actual weight of copper deposited was .570 grm.

What was the quantity of electricity which passed through the voltmeter? What was the average current? Was the ammeter correct?

7. A tangent galvanometer reads 32° with a current of .26 ampere. What is its reduction factor? What current would give double this deflection?

8. The reduction factor of a galvanometer is .87. What currents produce deflections of 10° , 20° , ..., 80° ? Draw a curve to connect deflection with current.

* Answers to examples are based on data in tables at the end of the book.

9. Find the current from the following readings: Weight of copper kathode 85.02 grms. before; 86.18 grms. after. The current was kept steady for 19 minutes.

10. Assuming the electro-chemical equivalent of silver to be .001118 gm., find that of copper from the following results of an experiment in which silver and copper voltameters were joined in series: Gain in weight of silver kathode = .755 gm.; gain of copper = .252 gm.

11. A current was passed through a water voltameter and a copper voltameter in series. The following results were obtained:

Volume of hydrogen (corrected to N. T. P.) 92.9 c.c.

Volume of oxygen 46.5 c.c.

Mass of copper deposited265 gm.

If the density of hydrogen is .0896 gm. per litre and of oxygen sixteen times as much, find the chemical-equivalents of oxygen and copper.

12. A current of 1.5 amperes flows through a copper voltameter with platinum electrodes. What substances are liberated and what amounts per minute?

13. How much oxygen is liberated per hour by a current of .5 ampere flowing through a water voltameter?

14. How many coulombs are required to liberate a litre of hydrogen?

15. If a metal cylinder (radius 2 cm., length 15 cm.) were rotated uniformly so that silver was plated evenly on its curved surface, find how long it would take to plate the surface with silver to a thickness of .01 mm. using a current of 1 ampere. Density of silver 10.6 grms. per c.c.

16. A metal plate, surface area 200 sq. cm., thickness 1 mm., is plated with the same metal so as to increase the volume 1%. A steady current of 1.5 amperes is passed through for an hour to effect this. Find the density of the metal if its E.C.E. is .00033.

17. What will be obtained at platinum electrodes when (1) dilute hydrochloric acid, (2) sodium chloride solution, (3) silver nitrate solution, (4) strong hydrochloric acid, are electrolysed?

CHAPTER III

THE VOLT

32. Mechanical Units. In our work so far we have introduced two new units, the coulomb and the ampere ; we are now going to introduce a third, the *Volt*. Before doing so, however, it may be well briefly to define a few terms used in Mechanics.

A force is said to do *Work* when the point of application moves in the direction of the force : work is measured by the product of the force and the distance moved in the direction of the force. Thus if by exerting a vertical force of 10 lbs. wt. I succeed in lifting a load through a height of 12 ft. the work done is 12×10 foot-pounds. In Electricity, however, these units of force and work are not convenient. The unit of force we use is the *Dyne*, the force that acting on a mass of one gramme produces an acceleration of one centimetre per second per second. Compared with the pound weight it is very small: roughly it is equal to the weight of a milligramme. The unit of work is the *Erg*, or dyne-centimetre. It is the work done by one dyne when the point of application moves a centimetre in the direction of the force. It is a very small unit: too small usually for practical work, so we multiply it by ten million and call the product a *Joule*. A joule ($= 10^7$ ergs) is approximately equal to the work done in lifting a kilogramme through ten centimetres. It is about $\frac{1}{4}$ foot-pound. The rate at which an agent works is called its *power*. The unit of power we use is the *Watt*. An agent works at one watt if its output is one joule per second. 746 watts are roughly equivalent to one horse-power.

Force, dyne ($= \frac{1}{981}$ grm. wt.).

Work, the dyne-centimetre or erg, 10^7 ergs = 1 joule.

Power, joule per second or watt, 746 watts = 1 horsepower.

33. Electricity and Energy. I remember looking up the word "Electricity" in a dictionary. All the information I obtained was that it was a "form of energy": and even that was wrong, for whatever electricity may be it certainly is not energy of any kind. Energy is capacity for doing work. Neither electricity nor matter is energy, but both may have energy. Electricity in motion—i.e. a current of electricity—has a certain kind of energy: matter in motion has also a kind of energy (kinetic energy). Electricity—as electricity—is almost useless. It is as a means of conveying, or directing, energy that electricity is employed all over the world to-day. It is necessary therefore to understand clearly the relation between the two.

If I buy water from a Water Company I may need it either as water, e.g. for washing, drinking, etc., or for the energy I can get from it. In the first case I shall be quite content to pay for it at so much per thousand gallons, and beyond satisfying myself of its purity shall need to ask no further questions. But the second case is not so simple; for example, suppose I wished to blow an organ with a hydraulic motor. Consulting an engineer's catalogue, I find that with water at 50 lbs. per sq. inch I need 12 gallons per minute; but at 30 lbs. per sq. inch I need nearly double as much. Of course then if I want water for driving machinery I shall not buy it by the gallon except on the understanding that the pressure of the supply is kept up to some definite amount. Then, though I speak of buying water, what I really buy is energy or work.

So with electricity. To light my house I employ electricity; but electricity alone is as plentiful as sea water and would be just as useful. It is energy that I really want, supplied electrically; and it is for energy that I pay. I may measure the energy I use by an electricity meter as I might measure the energy I took to work a hydraulic engine by means of a water meter: I am willing to pay so much per coulomb for electricity

or so much per gallon for water, but only because I know that each coulomb or each gallon is associated with a definite amount of energy.

The Electric Light Company actually supply me with electricity at fourpence a unit—but the unit is neither a coulomb nor any other unit of electricity. It is 1000 watt-hours. As a watt is a joule per second 1000 watt-hours must be $1000 \times 60 \times 60$ joules; hence I pay a penny for 900,000 joules.

34. To find the voltage of a supply. It is of interest to know how many coulombs are associated with each joule. It is quite easy to find this out as follows. Take an ordinary lamp and connect up with “flex” to the mains. One of the wires of the flex must be severed and the ends joined to an ammeter (or galvanometer) as shewn in fig. 24, so that you can find out what current the lamp takes. The lamp is to be inserted in a vessel of cold water. The water should be deep enough to cover the glass, but not to reach the adapter. It can be held down in position by a clamp. Take the temperature of the water and turn on the light for a measured time. Take out the lamp, find the new temperature of the water. Then weigh the water, and add to the result the water equivalent of the vessel. In this way you can get the heat delivered by the lamp in a known time.

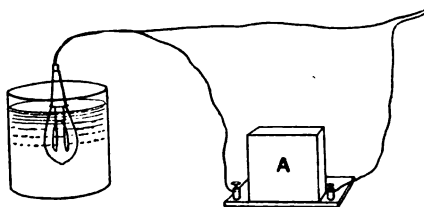


Fig. 24.

In an actual experiment the following readings were taken :

Current = .43 ampere.

Time = 240 seconds.

Therefore quantity of electricity used = 103 coulombs.

Initial temperature = 15°C .

Final temperature = 36°C .

Mass of water = 122 grms.

The vessel was an aluminium calorimeter; its water equivalent was 9 grms.

Heat absorbed = $(36 - 15)(9 + 122) = 2751$ calories.

Hence the supply is at the rate of 27 calories per coulomb.

Now heat is a form of energy: mechanical energy which we usually measure in joules can be transformed into heat energy, measured in calories. The rate of exchange is definite, 4.2 joules are equivalent to one calorie. Hence 4.2×27 , i.e. 113 joules, are supplied with each coulomb. This number measures what is called the *Voltage* of the supply. Briefly, **volts are joules per coulomb**. The above experiment was only a rough one, and the result not accurate. The supply is supposed to be at 105 volts.

The thing that is measured in volts is often called Potential Difference (P. D.), or Electromotive Force (E. M. F.). For the present we shall not try to define these terms. They are not exactly the same thing but there is no need yet to distinguish between them. Remember that the P. D. or E. M. F. between two points is measured by the energy developed per unit of electricity: and that volts are joules per coulomb. See also Art. 195 in Static Electricity.

35. Pressure and Work. The conception of voltage is often difficult to beginners: perhaps this is because we have no sense to detect it. We have a sense to detect pressure and temperature so that these two ideas present no difficulty. Suppose water is admitted to a cylinder fitted with a piston and that the piston area is one square foot. Every cubic foot admitted to the cylinder will drive the piston forward one foot and, if the pressure is p lbs. per square foot, exert a thrust of p lbs. weight on the piston: so that every foot of water will do p foot-lbs. of work. Hence to any one unfamiliar with the idea of pressure we might say that the pressure of still water is a quality in virtue of which it is able to do work; and that it is measured by the work per unit volume obtainable from it. This definition of pressure is not the usual one, but it is analogous to the definition of voltage as the work obtainable per unit of electricity.

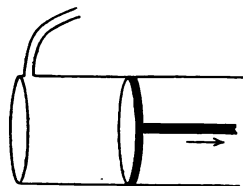


Fig. 25.

36. Voltmeters. An engine-driver must know his steam pressure: he reads it on a gauge which has been tested against a standard. So also an electrician needs some kind of gauge to measure voltage (or the number of joules associated with each coulomb); he could not possibly undertake an experiment like the above every time he wanted this information. The instrument in general use is called a voltmeter. There are several different kinds. The theory and mechanism of one form will be explained later. Meanwhile you will be expected to use them, just as you might use a steam gauge without troubling in the least to understand all about it.

Remember that a voltmeter is always connected up in parallel with the lamp, or wire, or motor, the voltage between the terminals of which is required: an ammeter is always in series.

The main current does not pass through a voltmeter any more than the steam passes through a steam gauge.

37. To make a galvanometer serviceable as a voltmeter. We shall suppose that the galvanometer has a radius of about 12 cm. and 20 turns and that it is required to register volts from about .5. With other ranges and sizes the details will differ. Take a Daniell cell and join it up to the terminals; the deflection will probably be well over 70° : in this case disconnect and insert in the circuit a resistance coil of such magnitude that the deflection falls to about 20° . A 20 ohm coil will probably be about sufficient. After the needle has become quite steady, read the deflection carefully: then change the battery terminals to reverse the current. Find the new reading and take the mean of the two as the true deflection.

Under the conditions of the experiment the E.M.F. between the poles of the Daniell will be about one volt. Now draw up a table according to the tangent law exactly as in Art. 30. This galvanometer in connection with the resistance may then be used to measure E.M.F. up to about 5 volts and so replace a voltmeter. It must not be regarded as an accurate instrument.

If you use a galvanometer with about 200 turns with a

resistance of two or three hundred ohms, you may assume that the E. M. F. indicated is 1.06 volts when it is in series with a Daniell. Such an instrument is much more accurate.

38. To find the voltage between the ends of a wire in which a current is flowing and to test the reading of a voltmeter. The method is in principle exactly like that of Art. 34, but this time the current will be taken not from high voltage mains but from a battery. Join up a few cells of a storage battery in series. Connect up in series (1) an ammeter *A*, (2) a coil of fine varnished wire *w* immersed in a calorimeter *C* half full of water. The wire in the calorimeter must be joined in parallel with the voltmeter, *V*.

It will be necessary to know

- (1) mass of water,
- (2) water equivalent of the calorimeter,
- (3) the initial and final temperatures,
- (4) the current,
- (5) the time during which the current flows,
- (6) the reading of the voltmeter when connected up as in fig. 26.

From (1), (2), (3) calculate the calories generated: multiply by 4.2 to get the number of joules. From (4) and (5) calculate the number of coulombs that have passed along the wire. From these two find the joules per coulomb, i.e. the volts between the ends of the wire.

(N.B. This result does not give the E. M. F. of the battery.)

Does the result agree with the voltmeter reading?

39. The power of a current. Suppose a current of *C* amperes flows along a wire and that the P. D. between the ends is *V* volts. The number of coulombs per second is *C*: and each coulomb is associated with *V* joules: hence the number of joules per second is *C.V*; but a joule per second is a watt: hence the watts of a current are calculated by multiplying the amperes by the volts.

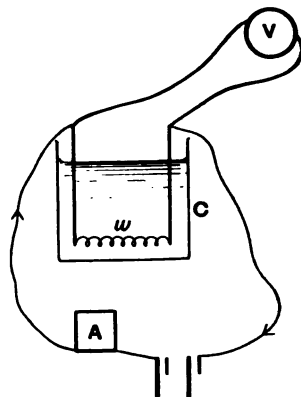


Fig. 26.

40. Motor Efficiency. Electricity is used not only for lighting and heating; it is also used for driving machinery. The voltage of a supply may be calculated by finding the work per coulomb that can be obtained from a motor: but it must be borne in mind that when any kind of energy is converted into mechanical work, some quantity is always lost, or rather dissipated in the form of heat. The ratio of the output of work from a motor to the intake of electrical energy is called the efficiency of the motor. The next experiment is designed to find the efficiency of a motor under a given load.

To measure the output of work we shall use a rope brake on the driving pulley. To one end of the rope is fastened a weight (*A*), to the other a spring balance (*B*). The direction of revolution must be as shewn. By knowing the weight at *A* and the reading of *B* we can find the pull of the rope at each side. Call the one *P* and the other *Q* lbs. weight. Measure the circumference of the wheel: *d* feet say.

Then the work done per revolution on the brake = $d(P - Q)$ foot-pounds.

Find by means of a revolution counter the number of revolutions per second. Of course this and all other readings must be taken while the motor is running on the brake. Call the number *n*, then $nd(P - Q)$ foot-pounds = work done in one second, but 1 foot-pound = 1.36 joules, hence the rate of working is $1.36 \times nd(P - Q)$ watts.

To find the power absorbed, the current (*C*) and the voltage (*V*) between the terminals of the motor must be read: the first by means of an ammeter, the second by means of a voltmeter. The product of the two readings measures the watts supplied. Hence the efficiency under the particular load

$$= \frac{CV}{1.36nd(P - Q)}.$$

In a small motor the efficiency is likely to be low, perhaps only five or six per cent. In a large machine it may be

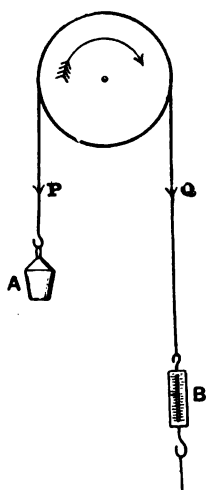


Fig. 27.

expected to be over eighty per cent. under its normal load.

41. Watts per candle power of an electric lamp.

The candle power may be found by means of any suitable photometer. The volts may be read on a suitable voltmeter by connecting the mains directly to its terminals. The current taken by the lamp is found as in the experiment of Art. 34. How does the result compare with the maker's advertisements?

42. Potential. A steam gauge is said to read pressure:

as a matter of fact it really reads difference in pressure. A gauge reading 60 lbs. per square inch indicates that the steam has a higher pressure than the air by 60 lbs. to the square inch. As the air pressure is about 15 lbs. to the square inch the actual pressure of the steam is about 75 lbs. per square inch. Some condensing steam engines take in steam above atmospheric pressure and eject it below: so to find the work which they ought to be able to do per pound of steam we require two gauges; though indeed a single gauge might be made to suffice if its inside could conveniently be connected to the supply and the outside be subjected only to the pressure of the exhaust. If we take the atmosphere as a standard, pressures above it are conveniently reckoned positive, pressures below negative: we have to remember that neither reading gives a true pressure.

Now all this applies in some degree to electricity. Though we sometimes speak of the "potential of a body" what we really mean is the potential difference between that body and the earth. (The gas or water-pipes form a convenient "earth.") We know approximately the pressure of the atmosphere, but it is not constant and it varies from place to place. The potential of the earth is likewise variable, though we cannot find its actual value anywhere. It is however a convenient standard from which to measure, for actual values are seldom required: it is the difference that is important. If we say then that a body has a potential of 50 volts we mean that the P. D. or E. M. F. between that body and the earth is + 50 volts; and that if the body were joined up by a wire to the earth a current would flow from the body to earth. If the potential

were -50 volts, the current would flow in the opposite direction: in either case the work per coulomb would be 50 joules.

43. Potential Fall and Potential diagrams. If water is flowing along a level pipe—say in the main of a long street—the pressure is not the same at all points, for some energy must be spent in making the water flow. Thus at one end the pressure may be 25 lbs. per square inch, at the other only 20. Take the case of water driven round a closed circuit by a pump (P); suppose it is used to drive a hydraulic

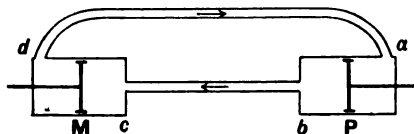


Fig. 28.

motor (M). The pump will deliver the water, say, at 25 lbs. per sq. in.: but the pressure will fall along the pipe bc and may be only 20 at c . The motor may absorb nearly all the energy conveyed by the water so that the pressure falls to 5 lbs. per sq. in. at d . The water then is returned to the pump again and is taken in at zero pressure at a .

The diagram shews the fall in pressure.

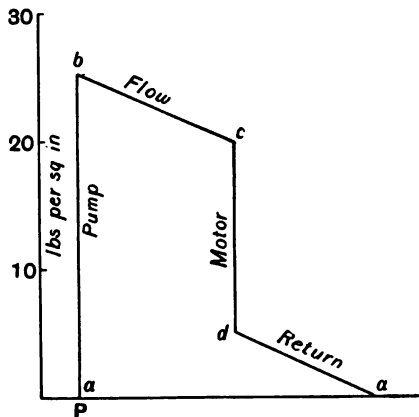


Fig. 29.

Something analogous takes place with electricity. Current produced by a dynamo or battery has to pass not only through lamps or motors but also through wires or "leads." Hence a dynamo if it is to supply lamps at 100 volts must deliver at a rather higher E. M. F. than this—say at 105. Fig. 30 is a potential diagram, intended to illustrate this case: there is supposed to be a fall of $2\frac{1}{2}$ volts along each lead.

If the leads are very thick and the current small the case is somewhat similar to that of water supplied through very big pipes and the fall in potential is small. In the next two diagrams the fall in the leads has been disregarded.

Let us suppose that four 25 volt lamps *A, B, C, D* are placed in series in a hundred volt circuit. A 25 volt lamp is one intended to give its normal candle power when the E. M. F. between the terminals

is 25 volts. A hundred volt circuit is one in which the main wires or leads have a potential difference of 100 volts: one may be at 100 volts (above the earth) and the other at 0, fig. 31; or one may be at 50, the other at -50 , fig. 32; or one at 0, the other at -100 . No electricity is used up in passing through the lamp, any more than water is used up in passing through a water engine. There is the same current at all

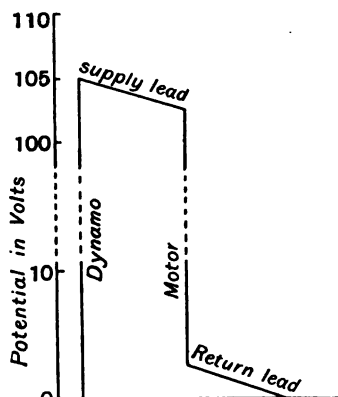


Fig. 30.

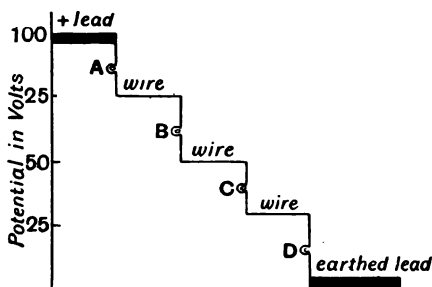


Fig. 31.

parts of the circuit. There is however a fall in voltage in passing each lamp ; in a similar way there is a fall in the pressure of water in passing through an engine. Some of the energy of the current is used up in the lamp ; some of the energy of the water is used up by the engine.

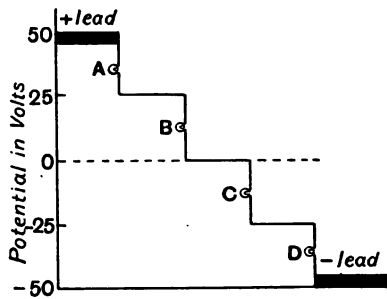


Fig. 32.

We shall conclude this chapter by placing in parallel columns some things in electricity which to some extent are analogous to other things in hydraulics.

Water	Electricity
Cubic foot or gallon or pound	Coulomb
Current	Current
Cubic foot per sec.	Ampere
Pressure	Potential
Difference in Pressure	E. M. F. or P. D.
measured in	
Lbs. per sq. ft.	Volts
Pressure gauge	Voltmeter
Pump	Dynamo
Water motor	Lamp or electromotor

EXAMPLES III

1. The burning of a kilogramme of coal liberates about 8 million calories. To how many kilowatt-hours is this equivalent?
2. A lamp takes a current of $\frac{1}{2}$ ampere, and the voltage between its terminals is 200. Find (1) the number of coulombs passed in one minute, (2) heat generated in one minute.
3. A lamp is immersed in a vessel containing 300 grms. of water. If it takes a current of 1 ampere at 50 volts, how much will the temperature of the water rise per minute?
4. Energy is sold by electric supply companies at a penny per kilowatt-hour (i.e. per "unit"). How much is this (1) per joule, (2) per calorie?
5. A motor takes a current of 10 amperes at 110 volts. What is (1) its power in kilowatts, (2) its horse-power?
6. What is the cost of boiling 30 gallons of water if energy is supplied electrically at 5d. per unit? Assume that the water is originally at 10°C . and that 10% of the heat supplied is wasted by radiation, etc. (1 gallon = 4530 c.c.)
7. A carbon incandescent lamp requires energy at 3 watts per candle power. If the price of energy is 3d. per unit, find how long a 16 c.p. lamp will burn for 1d.
8. A house is lighted by 45 lamps each of 16 c.p., each taking 1.2 watts per c.p. What H.P. is required to drive the dynamo?
9. In a house supply the voltage is 210. What quantity of electricity passes through the meter per hour when energy is taken at the rate of 150 joules per second?
10. An electric kettle holds two pints. It is stamped by the maker 210 v., 3 amp. In a test on 210 volt mains it was filled with water at 10°C .: it boiled after $12\frac{1}{2}$ minutes. The readings of the meter before and after use were 607.860 and 607.991 units. The price of electricity is a penny per unit. Find (1) cost per gallon of boiling water, (2) heat absorbed by the water, (3) percentage of heat wasted if the maker's stamp is correct, (4) if the readings are consistent.
11. A 16 candle power lamp is immersed in 250 grms. of water, the temperature of the water is found to rise 3°C . per minute. Calculate the watts per candle power.
12. A fine wire is placed in 500 grms. of water in a light calorimeter, and 5 amperes are passed through it, an E.M.F. of 14 volts being observed between the ends of the wire. Calculate the rise in temperature in 10 minutes.

13. Compare the cost of gas and electricity per candle power from the following data : The cost of electricity is $2\frac{1}{2}d.$ per unit ; a 70 c.p. gas burner consumes 5 c. ft. per hour ; price of gas, $2s. 6d.$ per 1000 c. ft. In electric lighting 1·1 watts are required per candle power.

14. What does the following statement mean ? The mechanical equivalent of heat may be taken as 427 kilogram-metres per kilogramme-calorie. Find the cost of boiling 10 kgrms. of water originally at $20^{\circ}C.$ with an electric supply at $3d.$ a unit.

15. If one gramme of hydrogen is burnt in oxygen, the heat generated is 34,500 calories. Find how many (1) calories, (2) joules, can be obtained by burning the gases liberated by the passage of one coulomb through a water voltameter.

(It follows from your answer that a certain minimum E.M.F. is required for the electrolysis of water.)

16. The following particulars of 4 volt accumulators are taken from a catalogue :

Ampere-hours	Weight in lbs.
10	$2\frac{1}{2}$
20	$5\frac{1}{2}$
40	$8\frac{1}{2}$
60	12
80	15

Plot a curve to connect the weight and the energy in joules and state how many joules are associated with a pound of lead in each case.

17. Energy is conveyed from a waterfall to a town some distance away by means of electricity. The dynamo delivers at 2050 volts, but in the town the E.M.F. is only 2000 volts so that there is a drop of 50 volts due to the leads. The current is 100 amperes. Find (1) the H.P. of the dynamo, (2) the percentage of energy wasted in the leads.

18. In an Edison accumulator the 300 ampere-hour cell weighs 25 lbs. and the discharge is at 1·2 volts. Find the energy stored per pound of material and find through what vertical height this energy could raise the cell.

19. Find the cost of boiling a quart of water in five minutes if electric energy is supplied at $2\frac{1}{2}d.$ per B.T. unit : the initial temperature of water being $16^{\circ}C.$ and the efficiency of the kettle 85 per cent. At what rate is the power supplied ?

20. Comment on :

“The heat equivalent of the watt is ‘24 calorie’ ;

“The energy from $\frac{3}{4}$ pint of petrol is equal to raising 900 tons one foot in one minute.”

21. Describe the experimental arrangements you would make if asked to determine the average value of an alternating current by means of the heat it develops in a coil of wire.

CHAPTER IV

RESISTANCE

44. Resistance. If the poles of a Daniell or Leclanché cell be joined by a long thin piece of iron wire, a very small current will pass along it: but if they be joined by a piece only an inch or two long, there will be a current which may be sufficiently large to make the wire red-hot. The E.M.F. is not greater in the second case than in the first so that the current that flows through a wire depends on something besides the E.M.F. The other factor on which it depends is termed *resistance*. We are going to deal with resistance in this chapter.

45. Relation between current and E.M.F. Take a length of wire—iron or German silver will do—and join it up to a single cell (Daniell, or storage) through an ammeter. Join the ends of the wire to the terminals of a voltmeter (fig. 33). Read the amperes and volts when the current is flowing.

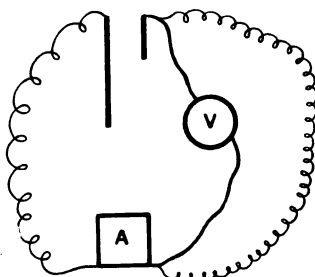


Fig. 33.

Now take two cells, connect them up in series and use them to drive a current through the same piece of wire. Take your readings again. Repeat with three cells in series and tabulate results.

	amperes	volts	volts ÷ amperes
1 cell			
2 cells			
3 cells			

A convenient way of varying the last experiment is to join up to the mains the ends of a long wire stretched on a frame: the wire on which you are experimenting may be joined to different points on the stretched wire. You can thus get a very large number of simultaneous readings of volts and amperes.

You will notice that the numbers in the last column of the table in which you write your results are all the same for the same wire. (Of course they will not be exactly the same; neither voltmeter nor ammeter is likely to be quite perfect; and you cannot measure small fractions of a volt or ampere.) This shews us that the E.M.F. between the ends of the wire is directly proportional to the current passing through it. This result was discovered by Ohm in 1826. We may state it in general terms and at the same time define what is meant by resistance.

46. Ohm's Law. If two places *A* and *B* are connected together, then the Electromotive Force between *A* and *B* bears to the current which passes between them a ratio which is always the same for the same connection under the same conditions. This ratio (E.M.F. to current) is termed the resistance of the connection.

Briefly, **resistance is E.M.F. per unit current.**

The resistance of a piece of wire is dependent on (1) the material, (2) the length, (3) the sectional area, (4) the temperature.

If water passes along a pipe, the rate of flow is dependent on the difference in pressure between the ends of the pipe: the greater the pressure the greater the rate, though the two are not exactly proportional. The flow is also dependent on the bore of the pipe and its length.

47. To find the resistance of a wire. To do this directly from the definition of resistance is merely a repetition of the experiment of Art. 45. The ratio E.M.F. to current gives us what is asked for. If we measure E.M.F. in volts and current in amperes the unit in which the resistance is measured is termed the Ohm. An ohm then is the resistance through which an E.M.F. of one volt will drive a current of one ampere. Briefly, **ohms are volts per ampere.**

48. The Ohm. This, however, is not the definition of the Legal Ohm. The Legal Ohm is almost exactly equal to it, but is defined as the resistance at 0°C. of 14.452 grms. of pure mercury in a column 106.3 cm. long and of uniform cross section.

49. Resistances in series. If several wires are joined in series, then the resistance of the whole is equal to the sum of their separate resistances. This may appear obvious; but we give a formal proof dependent on Ohm's Law.

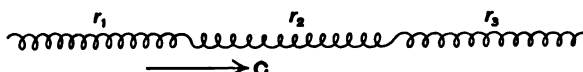


Fig. 34.

Suppose $r_1, r_2, r_3, \dots$ are the resistances of the separate wires, R the total resistance. Let a current C pass through them. By our definition of resistance we have

$$r_1 = \text{P. D. between ends of first resistance} \div C,$$

$$r_2 = \text{,, ,, ,, second ,,} \div C,$$

$$r_3 = \text{,, ,, ,, third ,,} \div C,$$

then by addition

$r_1 + r_2 + r_3 + \dots = \text{P. D. between extreme ends of the conductor} \div C$,
but the P. D. $\div$ current is defined to be the resistance, R , of the conductor,

$$\therefore R = r_1 + r_2 + r_3 + \dots$$

50. Resistances in parallel. Suppose two places A and B are connected by several different wires of which the resistances are r_1, r_2 , etc. Suppose currents c_1, c_2 , etc. flow along these wires and that the difference of potential between A and B is equal to E .

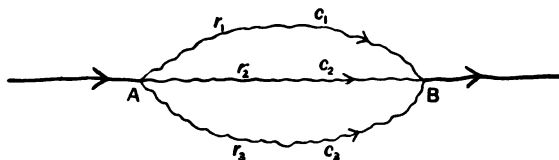


Fig. 35.

Then by Ohm's Law

$$c_1 = \frac{E}{r_1}, \quad c_2 = \frac{E}{r_2}, \text{ etc.}$$

Hence by addition

$$c_1 + c_2 + \dots = \mathbf{E} \left\{ \frac{1}{r_1} + \frac{1}{r_2} + \dots \right\},$$

but $c_1 + c_2 + \dots$ = the total current between A and B , and this divided by the difference of potential must be equal to the reciprocal of the total resistance ;

$$\therefore \frac{1}{\mathbf{R}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots$$

The reciprocal of the resistance is termed *conductivity*: hence the conductivity of the connection is equal to the sum of the conductivities of the parts.

The wires in this case are said to be arranged in *multiple arc* or *in parallel*.

51. Stranded Conductors. This result shews us that the resistance of a cable made up of several strands is inversely proportional to the number. Thus the resistance of $\frac{7}{2}$ —i.e. of seven strands of wire of 22 gauge—is one-seventh of that of a single strand the same length. The arrangement of several strands is adopted to make conductors flexible.

52. Shunts. If a conductor is divided into two or more branches, the magnitudes of the currents through the branches are directly proportional to their conductivities, i.e. inversely proportional to their resistances. In using measuring instruments it is often impossible to deal with the whole current,

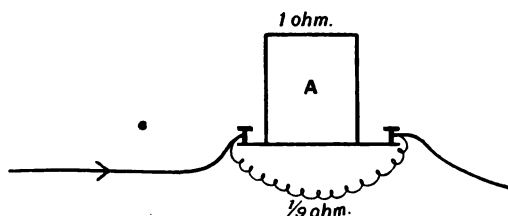


Fig. 36.

so that a galvanometer or ammeter is often shunted, that is to say its terminals are joined up through a resistance in such a way that the galvanometer and resistance are in parallel with one another. The main current then divides itself into parts proportional to the conductivities. Thus suppose the

resistance of the instrument is 1 ohm. If the terminals are connected by a wire of $\frac{1}{10}$ ohm resistance, nine times as much current will go through this shunt as through the galvanometer itself; i.e. only one-tenth of the whole current will pass through the galvanometer. The current indicated by the galvanometer must therefore be multiplied by 10 to give the current in the main circuit.

53. Specific Resistance. We often speak about the specific resistance of a material; by this we mean the resistance of a cube of one centimetre edge when the current enters in at one face and leaves at the opposite face and the direction of flow is everywhere perpendicular to these parallel faces.

If we know that a wire has a sectional area of α sq. cm., length l cm., resistance R ohms, we can find the specific resistance of the material—call this σ .

The resistance of a wire is proportional to its length; hence the resistance of a wire l cm. long and 1 sq. cm. in section would be $l\sigma$.

The conductivity of a wire is proportional to its cross sectional area; its resistance is therefore inversely proportional to its area. Hence the resistance of a wire l cm. long and α sq. cm. in sectional area would be $l\sigma \div \alpha$,

$$\therefore R = l\sigma/\alpha, \text{ i.e. } \sigma = \alpha R/l.$$

If a wire has a resistance R , diameter d , length l and the specific resistance of the material is σ , then $\sigma = \pi d^2 R/4l$.

If we measured specific resistance in ohms, the numbers for most metals would be very small indeed. It is usual therefore to select a smaller unit, the *microhm* which is one millionth of an ohm. It is in terms of the microhm that approximate values of specific resistance are given in the table at the end of this book.

54. Joule effect. The rate at which energy is spent—or heat developed—in the connection carrying a current between two points is proportional to the product of the E. M. F. and the current. Ohm's Law states that the E. M. F. is proportional to the product of resistance and current. It follows therefore that the heat must be proportional to the product of the resistance and the square of the current.

Suppose that a current of C amperes flows between two places A, B , that the E. M. F. between A and B is E volts and that the connection has a resistance of R ohms. Let the heat developed in t seconds be H calories.

In the time t the quantity of electricity which passes is Ct coulombs. The energy liberated is E joules per coulomb; i.e. ECt joules in all; by Ohm's Law

$$\frac{E}{C} = R,$$

$$\therefore \text{energy} = RC^2t \text{ joules.}$$

Now 4.2 joules are equivalent to one calorie,

$$\therefore 4.2H = RC^2t.$$

EXAMPLES IV

1. Two points are maintained by a battery at a constant potential difference of 48 volts: they are joined by three wires of resistances 6, 8, 15 ohms. What is the current in each wire?

2. What is the resistance of a wire which carries a current of 12 amperes when the E.M.F. between its ends is 3 volts?

3. In an electric light installation the mains are kept at a constant potential difference of 110 volts. They are connected together by 30 lamps, each taking $\frac{1}{2}$ ampere. What is the resistance of each lamp? The total current in the mains? The resistance of these 30 lamps in parallel?

4. Find the resistance (1) in series, (2) in parallel, of wires of resistances 1, $1\frac{1}{2}$, 2 ohms.

5. In parallel two wires have a resistance of $3\frac{2}{3}$ ohms; in series 14. Find the resistance of each.

6. Find the resistance per kilometre of a copper wire, $\frac{1}{2}$ cm. radius.

7. Find the resistance per mile of $7/22$ and $1/18$ copper cables.

8. Three points A, B, C are joined by wires. BC is 2 ohms, CA is 3 ohms, AB is 1 ohm. What is the resistance to a current of 3 amperes which enters at B and leaves at C ? What is the current in each wire? What is the E.M.F. between B and C and between A and C ?

9. The resistances of the sides AB, BC, CD, DA and of the diagonal AC of a quadrilateral are 1, 2, 3, 1 and 2 respectively. What is the resistance of the whole to a current entering at A and leaving at C ?

10. Two places, A and B , are joined by two wires of resistances 5 ohms and 3 ohms. The potential difference between A and B is maintained at 9 volts. Find the heat developed per minute in each wire.

11. Two wires are joined in series. If r_1 , r_2 are their resistances, find the ratio of the quantities of heat generated per second in them due to the passage of a current.

12. If two points—maintained at a constant difference of potential—are joined up by several wires, shew that the quantities of heat liberated per second are proportional to the conductivities.

13. What is the resistance of a 50 watt lamp on a 250 volt circuit?

14. A current of water flows along a metal pipe at the rate of 3 litres per minute. It enters at a temperature of 13.8°C . and leaves at 17.2°C . This rise is caused by the heat developed by a current of electricity which flows along the pipe. If the current is 12 amperes, find the e.m.f. between the ends of the pipe and the resistance.

15. A current of 1.2 amperes flows along a wire 6 metres long of resistance 8.5 ohms. Find the fall in potential per cm. along the wire.

16. A galvanometer has a resistance of 30 ohms: its terminals are shunted with a wire of resistance 5 ohms. What fraction of the whole current passes through the galvanometer?

17. With what resistance must a galvanometer of resistance 2 ohm be shunted if only .01 of the main current passes through the galvanometer?

18. How many amperes must be passed through a resistance of 100 ohms to convert 1 grm. of water per minute into steam?

19. A current of 2.1 amperes passes through a spiral of wire of resistance 10.2 ohms immersed in a liquid contained in a jacketed vessel. 4.2 grms. of the liquid boil away per minute. What is the latent heat of vaporisation of the liquid?

20. The poles of a cell are joined by two lengths of wire cut from the same reel: one wire is twice the length of the other. Which wire will be the hotter? In which is the greater heat produced?

21. The resistance of two wires in series is 36 ohms, in parallel 8.89. What is the resistance of each?

22. Two places are kept at a difference of potential of 15 volts. They are connected by three wires in parallel of resistances 2, 3, and 4 ohms. What is the current in each wire?

23. The resistance of a galvanometer is 25 ohms. With what resistance is it shunted if one-twentieth of the whole current passes through it?

24. A circuit, the total resistance of which is 20 ohms, has in it a 5 ohm coil. What must be the resistance of a shunt for the coil which will cause the heat generated in the coil to fall in the ratio of 3:1?

25. The difference of potential between the terminals of an incandescent lamp is 230 volts. The current taken is .25 ampere. How many ergs per second are used and how long will it be before sufficient heat is generated to raise the temperature of a litre of water ten degrees?

26. If 3 watts are required for each candle power in carbon lamps, what current does a 16 c.p. 100 volt lamp require?

27. Twelve wires, each of 3 ohms resistance, are joined up to form the edges of a cube. A current of .5 ampere flows in at one corner and out at the diagonally opposite corner. Find the current in each wire, the resistance of the frame and the P.D. between the opposite corners.

28. A current of 5 amperes passes through a wire of resistance .4 ohm. How many calories will be generated in an hour?

29. A wire is uniformly stretched by one-hundredth of its length. Assuming that the volume and specific resistance are unaltered, compare the resistances per metre before and after stretching.

30. Six wires are joined together so as to form a regular tetrahedron. The current flows in at one corner and out at another. Find the resistance of the network if each wire is of resistance 1 ohm.

31. Find from definition of the Ohm the specific resistance of mercury.

32. A square $ABCD$ is formed of a uniform piece of wire, and the centre is joined to the middle points of the sides by straight wires of the same material and cross section. A current is taken in at A and is drawn off at the mid-point of BC . Find the equivalent resistance if a side of the square is k ohms.

33. A, B, C, D are four points in succession at equal distances along a wire; and A, C and B, D are also joined by two other wires of the same length as the distances between these pairs of points measured along the straight wire. If a current enters the network thus formed at A and leaves it at D , shew that one-fifth of it passes along BC .

34. AB is a wire of uniform length and thickness (resistance 3 ohms) and is trisected at C and D . AC, CB are joined by wires of resistance 1 and 2 ohms respectively and BD, DA are joined by wires of resistance 1 and 2 ohms. The current enters at C and leaves at D . Find the resistance of the network.

CHAPTER V

THE E.M.F. AND RESISTANCE OF A CELL

55. Change of P.D. with current. We have hitherto made no distinction between Electromotive Force and Potential Difference. It is necessary now to define what we mean by the "Electromotive force of a cell." Perform the following experiment.

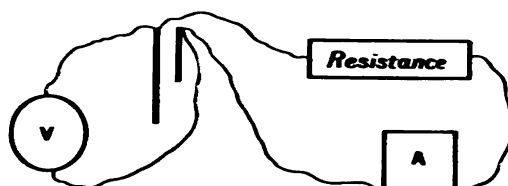


Fig. 37.

Take a cell and join its terminals to a voltmeter. Join them up also through an ammeter and a variable resistance in series (see fig. 37). Take a series of readings of the volts (E), amperes (C), and resistance (R).

E	C	R	CR
	0	infinite	

From this experiment you will see that the electromotive force between the poles of the cell (or the potential difference between them) is not constant, but is dependent on the current which the cell is giving: the greater the current, the less is the E.M.F. (or P.D.) registered by the voltmeter. By the "electromotive force of the cell" we mean the E.M.F. or P.D. between its terminals when the cell is not giving any current at all. Thus if we say that the E.M.F. of a Daniell cell is 1.1 volts, we mean that the E.M.F. (or P.D.) between the poles that would be registered by a voltmeter is 1.1 volts if the cell is not being used to give a current. If however the cell is giving a current, the E.M.F. of the cell is still 1.1 volts, but the E.M.F. (or P.D.) between its terminals will be less than 1.1 volts. (See also Art. 63.)

56. Hydraulic Model. It is easy to illustrate this fall in potential difference by a hydraulic model. *ABDE* is a pipe containing water. *C* is a tap, shewn closed in the diagram. *F* is a rotary pump. At *B* and *D* are vertical pipes, which act as gauges. The difference in the levels of the water in these pipes gives the difference in pressure between *B* and *D*.

If the pump is at rest, the water will be at the same height on each side: but if the wheel is being turned at a

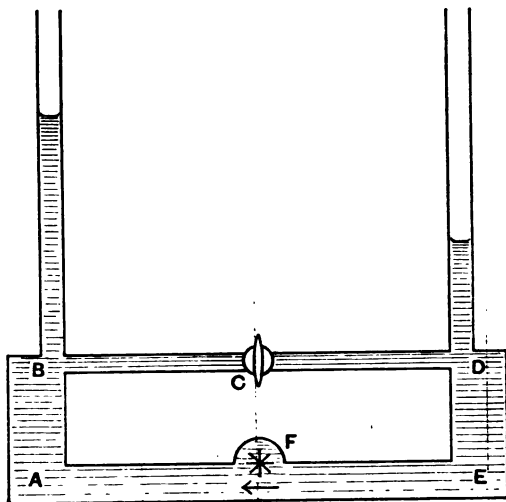


Fig. 38.

constant rate in the direction of the arrow it will tend to suck the water out of the right-hand half and push it into the left. The gauges will mark this and shew the water in the left-hand side higher than in the right. This pressure difference between the sides will remain constant as long as the wheel is turned at a constant rate, but if the tap *C* is closed, the machine can drive no current along *BD*. Now let the tap *C* be opened to some extent: the water begins to flow and we have a current from *B* to *D*. If the rate of turning still remains the same as before, this current will soon be steady and the water in the vertical tubes will take up new levels. *B* will be lower, *D* higher than before, shewing that the pressure difference between the sides has fallen. Further the more the tap is opened (i.e. the less the "resistance" of the circuit) the greater will be the current and the less the pressure difference between the sides. We might speak of the difference in level when no current is allowed to flow as the "motive force of the machine"; the difference in level, if a current is flowing, will always be less than this "motive force." It might be called the pressure difference or motive force between the sides of the machine.

57. Internal Resistance and Ohm's Law. Take the readings in the experiment of Art. 55, or better still take a new set. Let the resistance range from say $\frac{1}{2}$ ohm to 10 or 20 ohms. Measure the E.M.F. of the cell by a voltmeter. To do this it is only necessary to join the terminals of the cell to those of the voltmeter. The voltmeter is not required further in this experiment. Denote the E.M.F. by **E**. Take readings of the resistance (**R**) and the current (**C**).

R	C	$\frac{\mathbf{E}}{\mathbf{C}}$	$\frac{\mathbf{E}}{\mathbf{C}} - \mathbf{R}$

Now if you have had good instruments and have taken careful readings you will find that the numbers in the last column are constant. Call this constant **B**. We have then

$$\frac{E}{C} - R = B,$$

$$\text{i.e. } C(R + B) = E.$$

B is called the *resistance of the cell*: or we sometimes call it the internal resistance. **R** is the external resistance. The current which the cell gives has to pass through the complete circuit: i.e. it has to pass through the internal as well as the external resistance. We can now give **Ohm's law for a closed circuit**.

The ratio of the electromotive force of a cell to the current which it gives is equal to the total resistance of the circuit

$$E = C(R + B).$$

A convenient means of shewing the result of this experiment graphically is to plot a curve between the reciprocal of the current ($\frac{1}{C}$) and the external resistance (**R**).

Such a graph is shewn in fig. 39.

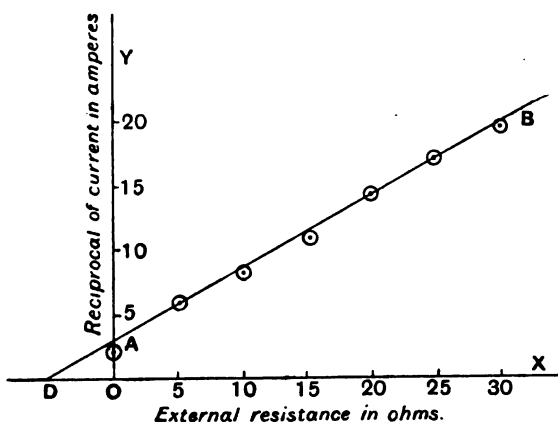


Fig. 39.

It is plotted from readings some of which are given in the table on the following page.

54 *The E.M.F. and Resistance of a Cell*

All the points lie nearly in the straight line AB . This line produced cuts OX in a point D , corresponding to 5.4 ohms; this must be the resistance of the cell for the graph shews that $R + 5.4$ is always proportional to $\frac{1}{C}$,

$$\text{i.e. } C(R + 5.4) = \text{constant.}$$

The constant measures the E.M.F. of the cell. It is equal to 1.8 volts.

If you use a tangent galvanometer, plot the resistance and the *cotangent* of the deflection: for the latter is proportional to the reciprocal of the current. There is no need to know the reduction factor to find the internal resistance.

C	$\frac{1}{C}$	R
.33	3.0	0
.116	8.6	10
.071	14.1	20
.051	19.6	30

58. To compare the E.M.F. of two cells. Call the E.M.F.s of the cells E, E' ; their resistances B, B' . Suppose we join up each in turn in series with a galvanometer or milliammeter and a resistance. Call the total external resistance R . We shall get currents C, C' which can be measured.

By Ohm's law we know $E = C(R + B)$; $E' = C'(R + B')$,

whence
$$\frac{E}{E'} = \frac{C}{C'} \times \frac{R + B}{R + B'}.$$

This relation would give us what we required if we knew R, B, B' . If these are not known, we can still get our result if R is very large compared with B and B' . Thus suppose $R = 1000$ ohms, $B = 1$ ohm, $B' = .5$ ohm.

Then
$$\frac{E}{E'} = \frac{C}{C'} \times \frac{1001}{1000.5} = \frac{C}{C'} \times 1.00005 = \frac{C}{C'} \text{ almost exactly.}$$

This shews that the E.M.F.s of cells are proportional to the currents which they drive through a large resistance.

In this experiment we do not require the actual current given by the cells: we only want to know the ratio of the first current to the second. There is therefore no real need to use

an instrument graduated in amperes (or in any subdivisions of an ampere). We could use a galvanometer of any kind in place of a milliammeter.

59. Voltmeters. An ordinary voltmeter is really an ammeter or galvanometer in which a high resistance is inserted. We could use a milliammeter with a resistance of 1000 ohms for finding the *E.M.F.* of a cell. For suppose a cell of *E.M.F.* **E** volts is joined up to this: the current produced would be $\frac{\mathbf{E}}{1000}$ amperes, i.e. **E** milliamperes. Hence every division on the scale would correspond to one volt. We have assumed here that the resistance of the voltmeter is great compared with that of the cell. In some cheap voltmeters this is not the case, so that the readings that they give for small cells of high resistance cannot be of much value.

EXAMPLES

1. A cell gives a current of 1.2 amperes when the external resistance is 1.5 ohms and of 2 amperes when the resistance is .7 ohm. What is the *E.M.F.* and resistance of the cell?

2. A battery drives a current of **C** amperes through an external resistance of **R** ohms. The values of **C** and **R** are given in the following table. Complete the table; plot a curve between $1/\mathbf{C}$ and **R** and so find the resistance and *E.M.F.* of the battery.

C	1.18	.95	.80	.69	.61	.54	.49
R	1	2	3	4	5	6	7
$1/\mathbf{C}$							

3. A milliammeter, resistance 2.5 ohms, graduated from 0 to 80 milliamperes is to be used as a voltmeter to register up to .8 volt. With what extra resistance should it be put in series?

4. A pocket voltmeter is correctly graduated but has a resistance of only 5 ohms. What will be its reading when attached to a Daniell of *E.M.F.* 1.08 volts and resistance .75 ohm?

56 *The E.M.F. and Resistance of a Cell*

5. A voltmeter of resistance 8 ohms is graduated from 0 to 5 volts. With what resistance should it be put in series if it is intended to register up to 150 volts?

6. A cell has an E.M.F. of 2 volts and a resistance of $\cdot 5$ ohm. Plot a curve to illustrate the relation between external resistance and the P.D. between the poles.

7. What current is produced by a cell of E.M.F. 1.5 volts and resistance $\cdot 5$ ohm in an external circuit of 3.75 ohms?

8. What E.M.F. is necessary to drive a current of 6 amperes through a wire of resistance of 1.5 ohms?

9. The voltage of a town supply is 230. A 16 c.p. lamp takes a current of .25 ampere. What is the resistance of the lamp?

10. A cell of E.M.F. 1.6 volts is used to ring a bell. The resistance of the cell is .2 ohm, that of the connecting wires 1.8 ohms, and that of the bell 1.5 ohms. What current will be produced when the circuit is closed?

11. A cell gives a current of 5.2 amperes through a resistance of .3 ohm. What is its resistance if its E.M.F. is 2.3 volts?

12. The readings of a voltmeter attached to the terminals of a cell are .96 and .8 when the readings of an ammeter in the circuit are 2 and 2.5. Find the readings when the total external resistance is .213 ohm. What is the resistance and E.M.F. of the cell?

13. A cell has a resistance of 2 ohms and an E.M.F. of 1.6 volts. What is the P.D. between its terminals when connected to a wire of resistance 6 ohms?

14. Find a relation to connect the current given by a cell, its resistance, E.M.F. and the voltage between its terminals.

15. What relation is there between the internal and external resistance in a circuit if the E.M.F. of the cell is 50 % greater than the P.D. between its terminals?

16. The terminals of a battery, of E.M.F. 4 volts and resistance 3 ohms, are connected by a wire of resistance 9 ohms. By how much is the P.D. between them reduced?

17. Find the current and the P.D. between the poles of a cell in the following cases:

(1) $E = 2.4$ volts, $B = .5$ ohm, $R = 3.1$ ohms;

(2) $E = 5.3$ volts, $B = 2.4$ ohms, $R = 10$ ohms.

18. A cell gives a current of .25 ampere in a coil of resistance 2 ohms. If its internal resistance is .2 ohm, find its E.M.F. and the P.D. between its poles.

CHAPTER VI

PRIMARY CELLS

60. Heat produced by action of acid on zinc. Take a test tube half full of dilute sulphuric acid and find its temperature. Weigh out some zinc filings: 2 grm. will be about the proper amount. Put the zinc into the acid. Chemical action takes place. Zinc sulphate is formed as the zinc dissolves and hydrogen is liberated. When the action has ceased read the temperature. Neglect the difference between the specific heat of the solution and that of water: neglect also the water equivalent of the test tube and the loss of heat that must take place during the reaction. Calculate the heat evolved in the experiment and thence the heat generated by the solution of one gramme of zinc in dilute sulphuric acid. If the zinc does not dissolve quickly the action may be accelerated by the addition of a few copper turnings or a *little* copper sulphate.

61. Simple Cell. Ordinary zinc is not pure: it contains small quantities of arsenic and iron and lead. It is said that quite pure zinc would not be acted on by the acid: such zinc is not easily procurable. If however ordinary zinc is first of all cleaned with dilute acid and then dipped into mercury, the two metals combine and form an amalgam. Amalgamated zinc is not readily attacked by dilute sulphuric acid.

Take a beaker of dilute sulphuric acid and dip into it a rod or plate of amalgamated zinc. Does any reaction take place? Now dip into the beaker a conductor (copper, platinum, carbon) which is not attacked by the dilute acid. Does this produce any effect?

Now join the zinc and copper together by a wire. What happens (1) at the copper, (2) at the zinc, (3) in the wire?

Test (3) by connecting through an ammeter and resistance and note the readings immediately the connection is made and see if it remains constant for the first half minute.

62. A Smee Cell consists of a plate of amalgamated zinc and a plate of silver roughened by a deposit of platinum. These are dipped into a vessel containing dilute sulphuric acid. The action is exactly like that of a simple cell, but the roughness on the silver enables the hydrogen formed to escape quickly when the circuit is completed so that the cell gives a more regular and continuous current.

63. The E.M.F. of a cell. If a gram of zinc is dissolved in acid a certain definite amount of heat is liberated. This quantity is independent of the rate at which the zinc is dissolved. It does not matter if the solution takes place in a flask or in a simple cell. There is however this difference. If the zinc is dissolved in a flask in the ordinary way, all the heat generated is liberated in the flask: if the zinc is dissolved in a simple cell while a current flows round the circuit, some of the heat is expended in warming the wire, the rest in warming up the cell.

Suppose the resistance of a simple cell is $\mathbf{B}$ ohms; let the poles be joined by a wire of $\mathbf{R}$ ohms. Suppose a constant current of $\mathbf{C}$ amperes flows round the circuit for t seconds. Then we know (Joule's Law) that the heat liberated in the wire is $\mathbf{RC}^2t \div 4.2$ calories; that in the cell is $\mathbf{BC}^2t \div 4.2$,

$$\text{total heat} = (\mathbf{R} + \mathbf{B}) \frac{\mathbf{C}^2t}{4.2}.$$

The total charge that passes in the t seconds is $\mathbf{C}t$ coulombs. Faraday's laws tell us that one coulomb of electricity is associated with .00034 grm. of zinc. Hence the zinc dissolved in the cell during the passage of $\mathbf{C}t$ coulombs is

$$.00034 \times \mathbf{C}t \text{ grm.}$$

Now we may find by a method similar to that of Art. 60 that the solution of one gramme of zinc gives rise to an evolution of 580 calories. Hence when .00034 $\mathbf{C}t$ grm. are dissolved we have $580 \times .00034 \mathbf{C}t$ calories.

If we may assume these two values of the heat to be equal, we get

$$580 \times .00034 \mathbf{C}t = \frac{\mathbf{R} + \mathbf{B}}{4.2} \mathbf{C}^2t,$$

$$\text{i.e. } (\mathbf{R} + \mathbf{B}) \mathbf{C} = .00034 \times 4.2 \times 580 = .83.$$

Now Art. 57 tells us that $(\mathbf{R} + \mathbf{B}) \mathbf{C}$ is the electromotive force of the cell. We expect then the E.M.F. of a simple cell

to be about .83 volt. The E.M.F. of most other cells is higher because further chemical actions take place.

We have this definition then: the electromotive force of a cell is measured by the total heat generated in the chemical action which must occur when a coulomb of electricity passes through the cell.

64. Polarisation. Take a simple cell: connect it up through a low resistance (say $\frac{1}{2}$ ohm) and ammeter: note the current and see if it remains constant. It is convenient to use a rod of amalgamated zinc in this experiment and a fairly large plate of copper. The whole experiment is often finished in a few seconds: but it may be repeated after the copper plate has been washed under the tap.

After a simple cell has been in use for a few seconds the current it gives is found to diminish considerably. The cell is then said to be polarised. This polarisation is due to the bubbles of hydrogen which collect all over the copper plate. These bubbles have two effects: they keep the solution off the copper and so increase the resistance of the cell; they also set up what is known as a back E.M.F. (*v.* Art. 24).

65. Daniell Cell. Several cells have been devised to get rid of polarisation and so obtain a constant source of E.M.F. In the Daniell the copper plate is surrounded by a strong solution of copper sulphate. The hydrogen instead of collecting on the copper acts on this solution, and is replaced by copper

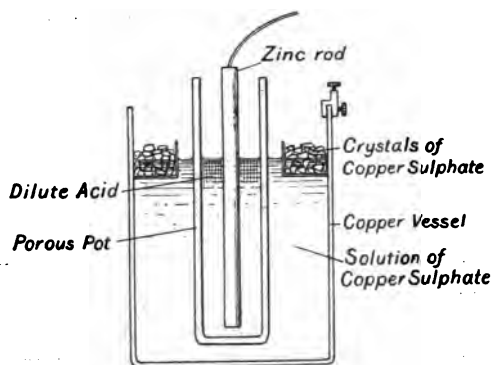
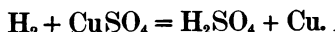


Fig. 40.

The copper is deposited on the copper plate. The strength of the solution would gradually decrease ; but to keep up the concentration crystals of the salt may be placed on a ledge round the top of the vessel. The earthenware vessel and plate of copper are often replaced by a copper vessel as in fig. 40.

Now zinc must not be placed in copper sulphate because the copper from the solution would be deposited on it. It is therefore necessary to keep the zinc and copper sulphate in separate compartments in the cell. This is conveniently done by having an inner porous pot. In this is placed dilute sulphuric acid and the rod of zinc. The pot must be porous: otherwise no current could pass through the cell. In the original cell made by Daniell in 1836 the "porous pot" was the wind pipe of an ox.

66. Grove and Bunsen Cells. In both these nitric acid is used as a depolarising agent. As this would rapidly dissolve copper, another substance must be found for the positive pole: it is platinum in the Grove and carbon in the Bunsen. A porous pot is necessary to keep the zinc out of reach of the nitric acid.

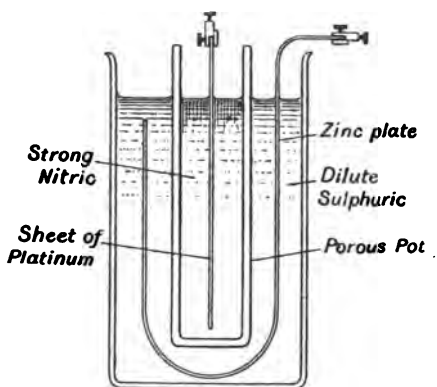


Fig. 41.

In the Grove (fig. 41) the sheet of platinum is usually placed in a narrow porous pot which holds the nitric acid: the outer vessel holds the zinc and sulphuric acid. In the Bunsen the zinc and sulphuric are usually placed in the porous pot; the outer vessel holds the carbon and nitric acid.

The cells have a high E.M.F. and low resistance but their use is restricted on account of the fumes which are liberated from the nitric acid, $\text{H}_2 + 2\text{HNO}_3 = 2\text{H}_2\text{O} + 2\text{NO}_2$.

67. The Chromic Acid Cell. The constituents of this are zinc, carbon, sulphuric acid, chromic oxide (CrO_3) or potassium bichromate. The last is the depolariser. No porous pot is necessary. The zinc plate is usually so arranged that it can be lifted up out of the acid when the current is not required.

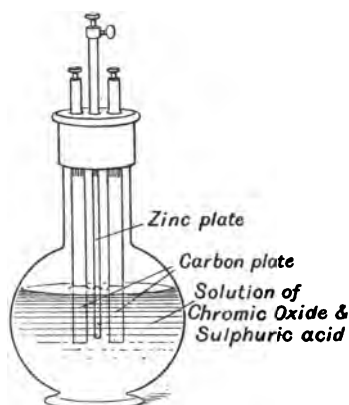


Fig. 42.

68. The Leclanché Cell. The poles of this are carbon and zinc; the depolariser is manganese dioxide: sulphuric acid is replaced by a solution of salammoniac. When the cell is in use, the zinc acts on the salammoniac, zinc chloride is formed and ammonia and hydrogen liberated,

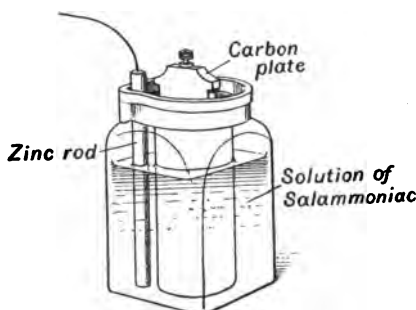
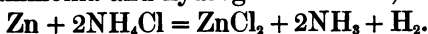


Fig. 43.

The ammonia escapes but the hydrogen unites with the manganese dioxide to form water and a lower oxide. The carbon plate is generally put in a porous pot: this is packed round with manganese dioxide and the pot sealed.

This cell quickly polarises so that it will not give a large current for any length of time. Its chief merit consists in the fact that it is not spoiled by standing, and that it rapidly recovers its normal voltage and resistance after use. It is admirably adapted for ringing electric bells, but would be quite unsuitable for lighting a lamp for more than two or three minutes.

69. The Dry Cell. This is much like the Leclanché. The positive pole, carbon, is placed in the centre. Round it is packed a damp paste of salammoniac, manganese dioxide, plaster of Paris. The outer vessel is made of zinc and acts as the negative pole. It is closed by a layer of pitch. This cell has the merits and defects of the Leclanché but the contents cannot spill.

70. The Clark Cell supplies us with a standard of electromotive force. It is not used to give a current and should never be put in circuit except with an extremely high resistance. It is usually of very small size: being set up in a test tube about two inches deep and an inch in diameter. The positive pole is mercury: this is joined to the terminal by a platinum wire. The negative pole, as usual, is of zinc. The mercury is at the bottom of the cell, above it a paste of mercurous sulphate, then a paste of zinc sulphate in which the zinc rod is immersed. The whole is covered by a layer of pitch.

The E.M.F. of the cell is 1.434 volts at 15° C. It falls about .001 volt for every degree rise in temperature. The cell is often fitted with a thermometer so that the E.M.F. may be corrected for rise or fall of temperature.

The **Weston Cell** supplies us with another standard of electromotive force. It is similar to the Clark, but the zinc and zinc sulphate of the latter are replaced by cadmium amalgamated with mercury and cadmium sulphate. Its advantage is that its E.M.F. of 1.019 volts is almost constant at ordinary temperatures.

71. Grouping of Cells. (1) *Series.* Suppose we have a battery of cells joined up in series: then the same current must pass through each of them. Now when a coulomb of electricity passes through a primary cell, '00034 grm. of zinc are necessarily dissolved. The heat generated by the solution of the zinc and by any other chemical action which may take place measures the E.M.F. of the cell. If then there are s cells in series, $'00034 \times s$ grm. of zinc per coulomb must be dissolved so that the E.M.F. of the battery must be s times that of one cell. The resistance is also s times that of one cell.

A battery then of 10 cells in series each of E.M.F. 1·4 volts and resistance '5 ohm would drive a current of $\frac{14}{15 + 5}$ ampere through an external resistance of 15 ohms.

(2) *In parallel.* If on the other hand we join up all the positive poles of p cells together to one terminal and all the negative to another and thus arrange them in parallel (or abreast); then when one coulomb passes through the external circuit, $\frac{1}{p}$ coulomb will pass through each cell so that the total zinc dissolved in the battery is only '00034 grm. per coulomb. In other words the E.M.F. of the p cells in parallel is only that of one cell. The current passing through the battery has p different paths: the resistance is therefore $\frac{1}{p}$ of that of a single cell. It is evident that a battery of little cells joined in parallel is equivalent to a single large cell.

A battery of 10 cells in parallel each of E.M.F. 1·4 volts and resistance '5 ohm would drive a current of $\frac{14}{'05 + '1}$ amperes through an external resistance of '1 ohm.

(3) *Mixed circuit.* Another method of grouping is to arrange the cells in rows, each row containing several cells in series. Thus suppose we have ps cells each of E.M.F. $\mathbf{E}$ and resistance $\mathbf{B}$ we could have p parallel rows, each row containing s cells in series. (In the figure there are twelve cells: they are arranged in three parallel rows of four.) The E.M.F. of the ps cells is evidently that of a single row—i.e. $s\mathbf{E}$. The

resistance of a row is $s\mathbf{B}$, the resistance of the p rows is therefore $\frac{s}{p}\mathbf{B}$. The current the battery will give through a resistance $\mathbf{R}$ is therefore

$$\frac{s\mathbf{E}}{\frac{s}{p}\mathbf{B} + \mathbf{R}}, \text{ i.e. } \frac{ps\mathbf{E}}{p\mathbf{R} + s\mathbf{B}}.$$

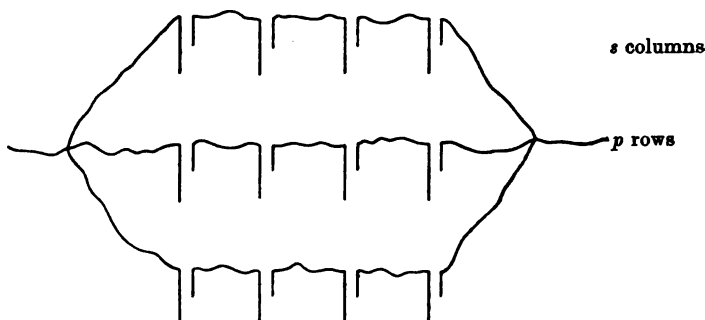


Fig. 44.

72. Maximum Current. With a given number of cells what arrangement should be made to obtain the maximum current through a given external resistance? If the external resistance is very large, the cells should be arranged in series: if the external resistance is very small, they should all be joined abreast.

The third grouping is useful when the external resistance is comparable with that of the cells. If we have n cells, where n can be factorised, our problem is to find the factors p and s which make $\frac{ps\mathbf{E}}{p\mathbf{R} + s\mathbf{B}}$ a maximum. This expression is a maximum when the denominator $p\mathbf{R} + s\mathbf{B}$ is a minimum. Now the product of $p\mathbf{R}$ and $s\mathbf{B}$ is fixed ($= n\mathbf{R}\mathbf{B}$); their sum is therefore least when they are equal to one another. (A rectangular figure of given area has least perimeter when it is a square.) We get then the maximum current if we can make $p\mathbf{R}$ equal to $s\mathbf{B}$. An example will make this more evident.

Suppose we have 24 cells each of E.M.F. 1.5 volts and resistance 1.2 ohms: how should they be arranged to give the maximum current through an external resistance of .45 ohm?

Divide the 24 cells into p groups, each consisting of $s \left(= \frac{24}{p} \right)$ cells joined in series. Then the E.M.F. of each group is $1.5s$ volts, the resistance $1.2s$ ohms. If the p groups are joined in parallel, the E.M.F. of the battery so formed is still $1.5s$ volts, but the resistance is only $\frac{1}{p}$ of $1.2s$.

Hence

$$\text{current} = \frac{1.5s}{1.2 \frac{s}{p} + .45} = \frac{1.5ps}{1.2s + .45p} = \frac{24 \times 1.5}{1.2s + .45p} \text{ amperes.}$$

To make this as large as possible we must so arrange p and s that $1.2s + .45p$ is as small as possible. This is done by putting $1.2s = .45p$,

$$\text{i.e. } 8s = 3p.$$

But $ps = 24$, so that we must have $s = 3$, $p = 8$.

Substituting, we get that the maximum current

$$= \frac{1.5 \times 24}{3.6 + 3.6} = 5 \text{ amperes.}$$

EXAMPLES

1. Why is the simple cell unsuitable for general use?
2. How many cells, each of resistance .5 ohm and E.M.F. 2 volts, would be required to drive a current of 1 ampere through a lamp of resistance 25 ohms?
3. Volta's pile consisted of a series of discs of copper, zinc, and flannel moistened with dilute acid, arranged in the following order: copper, zinc, flannel; copper, zinc, flannel, etc. Why would not this give a large current? For what purposes could it be used?
4. A storage cell of E.M.F. 2.3 volts is connected up to two circuits in parallel. In the one is a voltmeter of very high resistance which registers 1.9 volts; in the other is a resistance coil placed in series with an ammeter of negligible resistance which registers 4.75 amperes. Make a diagram of the apparatus. Find the resistance of the coil and of the cell, and the rate at which the cell is working.

5. Eight cells, each of E.M.F. 1 volt and the resistance .5 ohm, are arranged in series, and the terminals joined by a wire of 4 ohms resistance immersed in 100 c.c. of water. The current is passed for five minutes. Find the rise in temperature of the water.

6. Can two pieces of the same metal immersed in a solution produce a current? Explain the current produced when the platinum electrodes of a voltmeter, recently in use, are joined up to an ammeter.

7. What is the function of a porous pot in (1) the Daniell cell, (2) the Grove cell? Why must the pot be porous?

8. Two vessels, one containing a copper plate and a solution of copper sulphate, the other a plate of zinc and dilute sulphuric acid, are connected by an inverted U tube filled with dilute acid. The plates are joined by a wire. Explain carefully what takes place in the wire, in the tube, and in the two vessels?

9. Explain fully why the bubbles on the copper plate of a simple cell affect the current.

10. Can you give any reasons why the common primary cells have zinc for the negative pole? Could a cell with iron or magnesium be used?

11. Why should a Clarke cell never be used to give a current?

12. How is the efficiency of a cell affected by increasing the size? Is there any difference in the currents given by cells of different sizes when the external resistance is, say, 1000 ohms?

13. A current of one ampere flows for 10 minutes through a Daniell cell. How much zinc will be consumed and how much copper will be deposited?

14. State what chemical action takes place in (a) Leclanché, (b) Grove, (c) Daniell.

15. Compare the relative usefulness of (a) batteries of primary cells, (b) accumulators, (c) dynamos, for generating current.

16. Six cells each of E.M.F. 2 volts and resistance 1 ohm are used to drive a current through an external resistance of 5 ohms. Find the currents produced when the cells are (1) in series, (2) in parallel, (3) two parallel sets of 3 in series.

17. What relation must there be between the external resistance and the cell resistance, if two cells give the same current when connected in parallel and in series?

18. Find the current which two cells of E.M.F. 1.08 volts and resistance .5 ohm will drive through a resistance of .3 ohm (1) when in series, (2) in parallel.

19. A battery of 6 cells is arranged in mixed circuit so that there are 3 rows containing respectively 1, 2, and 3 cells. If the E.M.F. of each cell is 2 volts, and resistance 1 ohm, find the current through an external resistance of 12 ohms.

20. Find the currents given by different arrangements of 6 cells, each of resistance 1 ohm, E.M.F. 1 volt, with an external resistance of 5 ohms.

21. You are given 48 cells, each of E.M.F. 1·8 volts and resistance ·5 ohm. Find the greatest current they will produce in a circuit of 1·5 ohms external resistance.

22. Given 16 cells, each of resistance ·5 ohm and E.M.F. 1·3 volts, how would you arrange them to get the maximum current through

- (1) an external resistance of 100 ohms ;
- (2) a copper rod of resistance ·01 ohm ;
- (3) a resistance of 2 ohms ?

Find the current in each case.

23. Three storage cells, each of E.M.F. 2 volts and negligible resistance, are joined in series, and the poles of the battery so formed are connected by a coil of 6 ohms and a coil of 12 ohms in parallel. Find (a) the current in each coil, (b) the heat developed per second in each coil.

24. Three cells, each of E.M.F. 1·5 volts, are to be arranged either in series or in parallel. The internal resistances of the cells are ·4, ·5, ·45 ohm respectively. Which arrangement will give the larger current through an external resistance of (1) ·1 ohm, (2) 2·1 ohms ?

25. A circuit is formed of six similar cells in series and a wire of 10 ohms resistance. The E.M.F. of each cell is 1 volt, and its internal resistance 5 ohms. Determine the difference of potential between the positive and negative poles of any one of the cells.

26. A battery of 12 similar cells in series screwed up in a box, being suspected of having some of the cells wrongly connected, is put into circuit with a galvanometer and two cells similar to the others. Currents in the ratio of 3 to 2 are obtained according as the introduced cells are arranged so as to work with or against the battery. What is the state of the battery ?

27. A battery of 12 cells in series, each of E.M.F. 1 volt and resistance ·5 ohm, is joined up to a coil. What must be the resistance of this coil if the heat produced in it is a maximum ?

CHAPTER VII

BRIDGE METHODS

73. Wheatstone's Bridge. Suppose we have four points— A , B , C , D —which are connected together in the following way: A and B by a resistance $\mathbf{R}$, A and D by a resistance $\mathbf{X}$, B and C by a resistance $\mathbf{S}$, D and C by a resistance $\mathbf{K}$. Let also A and C be connected to the terminals of the battery, B and D to the terminals of a galvanometer, G .

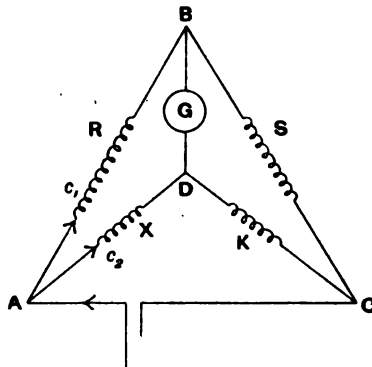


Fig. 45.

Now by altering the resistances $\mathbf{R}$ and $\mathbf{S}$ we can cause the current to flow in either direction through the galvanometer, or, we can so adjust them that no current flows through at all. In this case all the current which flows along AB must continue to flow along BC , and that along AD must continue along DC . Call these currents c_1 , c_2 .

Now there is no current from D to B , so that there can be no P.D. between these points: therefore the P.D. between A and D must be the same as that between A and B ,

$$\text{i.e. } c_1 \mathbf{R} = c_2 \mathbf{X}.$$

By exactly similar reasoning

$$c_1 \mathbf{S} = c_2 \mathbf{K}.$$

Hence by division

$$\frac{\mathbf{R}}{\mathbf{S}} = \frac{\mathbf{X}}{\mathbf{K}},$$

or

$$\mathbf{XS} = \mathbf{RK}.$$

Hence the resistance of one arm can be found in terms of the other three.

Perhaps it may be well to look at the method from another point of view.

A current flows from A to D through $\mathbf{X}$ and then continues to C through $\mathbf{K}$. Hence the fall in potential between A and D is to the fall between D and C as the resistance $\mathbf{X}$ is to the resistance $\mathbf{K}$. A similar relation holds for the arms AB , BC .

Now the potential at B is the same as the potential at D for no current flows between these points. We have then

$$\begin{aligned} \mathbf{X} : \mathbf{K} &= \text{fall between } A \text{ and } D : \text{fall between } D \text{ and } C \\ &= \text{fall between } A \text{ and } B : \text{fall between } B \text{ and } C \\ &= \mathbf{R} : \mathbf{S}. \end{aligned}$$

74. B.A. Bridge. The B.A. (British Association) bridge method of finding resistance is dependent on this principle. Fig. 46 represents the apparatus which consists of a known resistance $\mathbf{K}$, a uniform wire $\mathbf{RS}$, a sensitive galvanometer G and a battery. $\mathbf{X}$ is the unknown resistance. The thick

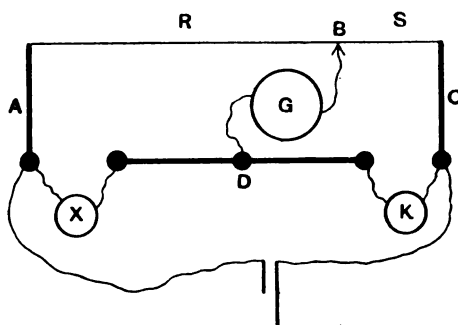


Fig. 46.

lines indicate bars of brass or copper having no appreciable resistance. The end (*B*) of the wire leading from the galvanometer is not fixed but is joined to a jockey which must be moved up and down the wire **RS** until a point is found which is such that no current passes through the galvanometer on depressing the key.

The resistances of the two parts of the wire **RS** are proportional to their lengths. Applying the results of the previous paragraph we see that the ratio of the resistance **X** to **K** must be the same as the ratio of the lengths of the two parts of the wire,

$$\text{i.e. } X = \frac{R}{S} K.$$

In making an experiment it is advisable to verify the result by interchanging the position of the known and the unknown resistances. Assume the mean of your result to be correct. The known resistance (**K**) should be chosen approximately equal to what you think **X** is likely to be.

75. P.O. Box method of finding Resistance. The P.O. box contains a series of resistances arranged as in fig. 47.

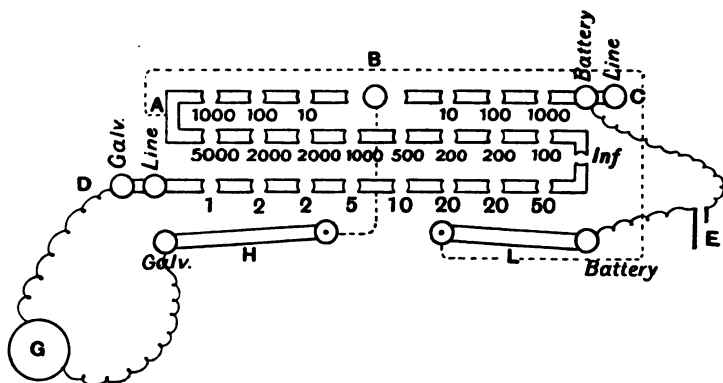


Fig. 47.

In the top row, *A, B, C*, there are generally six resistances, 1000, 100, 10, 10, 100, 1000: these correspond to the ratio arms (**R, S**) in the wire bridge (Art. 74). The point *B*, which

is the junction of the two sets of three, is joined by a connection (shewn dotted) to the terminal *D* through the key *H* and the galvanometer *G*. The terminals *C* and *D* are joined to those of the unknown resistance **X**. The known resistance **K** consists of a series of coils: at its ends are terminals *A*, *D*. *A* and *C* are connected through a key *L* and a cell *E*.

Now suppose you wish to find the resistance **X** of a coil. The apparatus is arranged as in the diagram. Feel the plugs and see that they are all just tight in their conical holes. Pull out the two "tens" in the top row: press the galvanometer key *H* and tap instantaneously the battery key *L*. There will be a small motion of the galvanometer: suppose the throw is to the right. This means that if the known resistance **K** is smaller than **X**, the throw is to the right.

Now pull out from **K** a plug corresponding to a resistance which you believe larger than **X**. Suppose this is 300. Tap: if the throw is to the left, then the resistance of **X** is between 0 and 300. Now replace the 300 plug, and pull out say the 100. If the throw is to the right, **X** is bigger than 100. We may proceed until we find two consecutive numbers between which the resistance of the coil must lie. We can in this way find correct to an ohm the resistance of any coil between 1 and 10000 ohms.

The further working will be best explained by an actual example in which the result finally obtained was 5·83 ohms. Starting off exactly as before it was found that 5 ohms in **K** gave a deflexion to the right, 6 to the left. To obtain the next figure 8, the resistance in ratio arm **S** was altered by replacing the ten plug and removing the 100. Since **S** is now ten times **R**, **K** must be ten times **X** if we are to obtain a balance. It was found by experiment that 58 ohms in **K** gave a deflexion to the right, 59 to the left. The resistance of **X** was therefore between 5·8 and 5·9 ohms.

Now the 1000 in **S** was removed and the 100 replaced. With 584 ohms in **K** there was a deflexion to the left: with 583 there was a very slight deflexion to the left. The resistance of **X** was therefore just greater than 5·83 ohms.

If it had been required to find the resistance of a coil of say 52,000 ohms, we should have had to use the larger resistance in **R** instead of **S**.

76. The resistance of an electrolyte cannot be determined by any of the above methods, for currents passing through electrolytes decompose them and there is a back E.M.F. to be considered. A method often employed involves the use of a B.A. bridge, an alternating current and a telephone receiver. The alternating current avoids the polarisation, for the rapidity of the reversal of the direction prevents any appreciable quantity of the ions accumulating on the electrodes. The telephone replaces the ordinary galvanometer which only detects direct currents. When a balance between the arms of the bridge is obtained the telephone is the least noisy. The comparison resistances must be non-inductive (Art. 137).

77. Galvanometer Resistance: Kelvin's method. Place the galvanometer in the "unknown-resistance" gap of a B.A. bridge. The middle terminal must be connected directly to the jockey and the battery placed between the other two terminals.

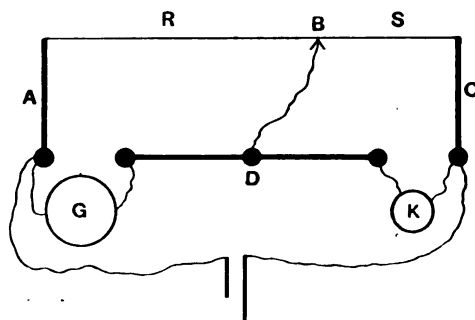


Fig. 48.

There will always be a deflection in the galvanometer, but when the jockey is in the right position this deflection will not be altered by depressing the jockey key.

The explanation follows directly from Art. 73. For suppose that the position of the wire BD is found such that no current flows along it on making contact at B ; then there can be no alteration in the previous currents in other parts of the apparatus and hence no change in the reading of the galvanometer in the resistance gap.

78. Battery Resistance: Mance's method. Place the comparison coil K in the "known-resistance" gap, CD . Place the battery E in the unknown resistance gap BD .

Place the galvanometer G in the galvanometer gap BD . Place a tapping key k in the battery gap AC .

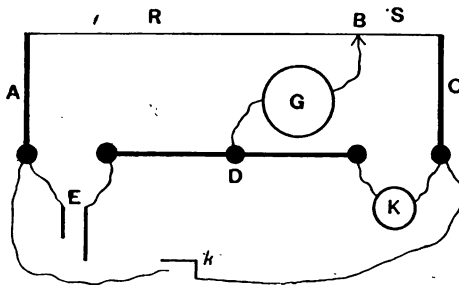


Fig. 49.

The jockey at B must be depressed the whole time. There will always be a deflexion in the galvanometer. The correct position of B is such that there is no immediate change in the deflexion of the galvanometer when the tapping key (k) is depressed.

The following reasoning though not quite satisfactory will be sufficient to explain the theory of Mance's method. When the conjugate condition is satisfied (i.e. $SB = RD$), the addition or alteration of any source of E.M.F. in the arm AC would not affect in any way the current in the arm DGB . Now the closing or opening of the key k changes the p.d. between A and C : this then will only fail to affect the current in G if the conjugate condition is satisfied.

79. The Carey Foster Bridge is a modification of the B.A. bridge and is used to find the difference between two nearly equal resistances. In fig. 50, X and Y are these resistances; H, K another pair which should also be nearly equal though their actual values are not required. AC is a wire of uniform resistance, G a galvanometer, E a cell. The arrangement then only differs from the B.A. bridge in having the extra gaps in X and Y .

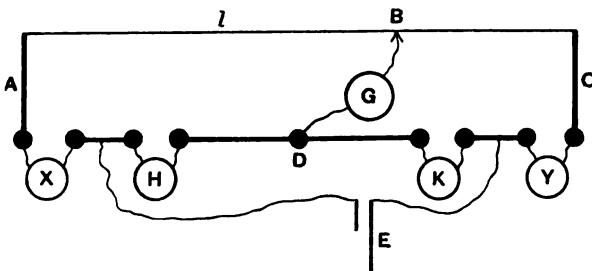


Fig. 50.

Suppose B is the position for the jockey on the bridge wire when no current passes through the galvanometer: then the conjugate condition must be satisfied, i.e.

$$H:K = \text{resistance of } X \text{ and } AB : \text{resistance of } Y \text{ and } BC.$$

In other words the total resistance of X , Y and AC is divided at B in the ratio H/K .

Now interchange X and Y : suppose B' is the new position of the jockey: then as before the total resistance of Y , X and AC is divided at B' in the ratio H/K ; hence the resistance of X and AB must be equal to the resistance Y and AB' , so that the difference between X and Y must be equal to the resistance of the intercept BB' on the bridge wire. To find this resistance, measure in any way the total resistance of the wire and multiply it by the ratio BB'/AC .

CHAPTER VIII

THE POTENTIOMETER : KIRCHHOFF'S LAWS

80. The Potentiometer. If a steady current flows along a wire of uniform resistance there must be a uniform fall of potential from one end to the other: or in other words the potential difference between any two points on the wire must be proportional to their distance apart.

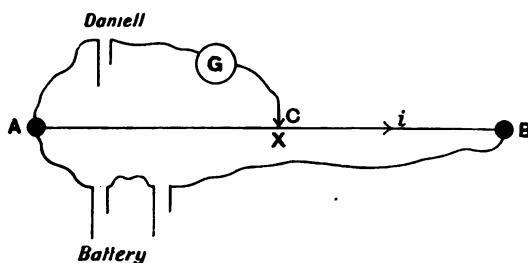


Fig. 51.

In the diagram (fig. 51) a battery of cells is joined up to a long uniform wire AB ; positive pole to A , negative to B . The terminal A is also joined up to the positive pole of another cell, say a Daniell. The negative of this cell is connected to a galvanometer. The free end C of the wire leading from the galvanometer can be joined up to make contact with any point on the uniform wire AB . As long as it is not connected up to the wire, the potential difference between A and C must be equal to the E.M.F. of the Daniell, for no current can be flowing through this cell. Now suppose X is a point on AB such that the fall in potential between A and X is equal to this potential difference: in other words X is a point on AB such that the fall in potential between

A and X along the uniform wire is equal to the fall between A and C through the Daniell cell. Then X and C must be at the same potential so that no current would flow between them if they were united. If then this point X can be found such that no current flows through the galvanometer when the circuit is completed, the fall in potential between A and X must be equal to the E.M.F. of the Daniell cell.

Now let the Daniell be replaced by another cell; say a Leclanché: a new point Y may be found corresponding to X . We shall then know that the potential fall along AY is equal to the E.M.F. of the Leclanché.

Hence

$$\frac{\text{E.M.F. of Daniell}}{\text{E.M.F. of Leclanché}} = \frac{\text{P.D. between } A \text{ and } X}{\text{P.D. between } A \text{ and } Y} = \frac{\text{length } AX}{\text{length } AY}$$

In laboratory potentiometers the wire AB is usually mounted on a board in zig-zags: a sliding bridge fitted with a tapping key enables any point on the wire to be connected to the cell under test.

81. To compare E.M.F. or find Voltage. Take an accumulator or battery of rather higher E.M.F. than that of either of the cells to be compared, and join its positive pole to one end (A) of the potentiometer wire. Join the positive pole of one of the cells to the same end (A). Connect up the negative pole of the cell through a sensitive galvanometer to the bridge key and the negative pole of the accumulator to the other end (B) of the potentiometer wire. Move the bridge about until you find a point on the wire such that no deflexion is produced in the galvanometer by depressing the key and making contact. Measure the distance of the point from the end A .

Replace the first cell by the second and proceed as before.

To find the voltage of a cell compare the E.M.F. with that of a known cell: say the standard Clark (1.435 volts at 15° C.).

For low voltages the modification described in Art. 101 is useful.

For high voltages we may connect up in the method shewn in fig. 52. We will suppose that A and B are points on the mains of a lighting circuit, the voltage of which is known to be about 200. If the resistances of R_1 , R_2 are 4950

and 50 ohms respectively then the potential difference between the terminals of R_2 is one hundredth of the potential difference between AB and therefore about 2 volts. If then we find exactly by use of the potentiometer in the ordinary way the potential difference between the terminals of R_2 and multiply by 100 we get the result required.

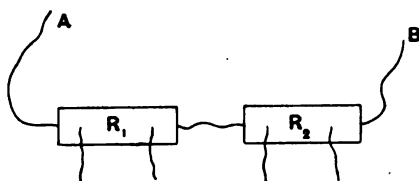


Fig. 52.

82. To compare the resistances of two coils by means of the potentiometer, a *constant* current is passed through the two in series, fig. 52. The potential difference between the ends of each resistance is then found by joining the resistance terminals up to the potentiometer in exactly the same way as the cells were joined up in the preceding paragraph.

The ratio of the potential differences in the two cases must be the same as the ratio of the resistances of the coils.

If no bridge is supplied with the potentiometer, a convenient method of making contact is to insert the loose end of the wire into the hinge of a knife and place the *back* of the blade on the wire.

83. Extension of Ohm's Law. Suppose a cell of electromotive force $\mathbf{E}$, internal resistance b , gives a current c through an external resistance x . Then the potential difference between its poles is, by Ohm's law, cx . We also know that $\mathbf{E} = c(x + b)$. Hence when a current c passes through a cell of electromotive force $\mathbf{E}$ and resistance b , the potential difference between its terminals is $\mathbf{E} - cb$.

Let two points PQ be joined together by wires of total resistance r and a cell the E.M.F. of which is $\mathbf{E}$ and the resistance b . If they are not connected in any other way, of course no current can flow: let them, however, be connected in some way—say by a wire or by a second cell—and let the

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current which now flows be denoted by c . Then, as we have seen above, the potential difference between the poles of the cell must be $\mathcal{E} - cb$.

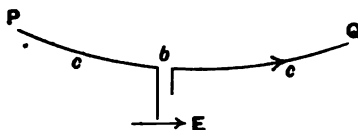


Fig. 53.

The current c flows also through the resistance r : the drop in potential therefore between P and Q due to this resistance is cr : subtract from this the rise in potential through the cell and we get the result that the total potential difference between P and $Q = cr - (\mathcal{E} - cb)$,

i.e. the P.D. between P and $Q = c(r + b) - \mathcal{E}$.

Hence if two points P and Q be connected by a conductor the potential of P above Q together with the electromotive force of any cell in this connection is equal to the product of the current and the total resistance.

Or if p and q denote the potentials of the points P and Q , $p - q + \mathcal{E} = CR$.

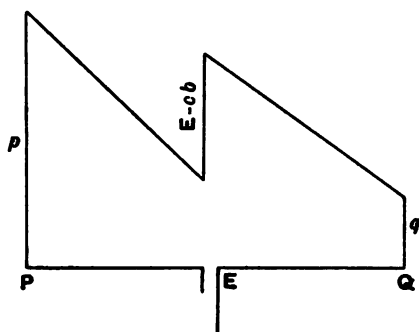


Fig. 54.

In this formula the current is supposed to be from P to Q : the cell tends to drive in the same direction. If either of these is reversed the sign of the corresponding term must also be reversed.

84. An example will make the use of the formula clear:

The positive poles A, A' of two cells of E.M.F. 2 volts, 3 volts and resistance 2, 1 ohms are joined by a wire AXA' of resistance 8 ohms, and the negative poles B, B' by a wire BYB' of resistance 6 ohms. X, Y are the middle points of the wires; find the p.d. between X and Y .

Suppose the current is c and that it flows in the direction of the arrow on the wires: the arrow on each cell denotes the direction of the E.M.F. of that cell.

The total E.M.F. of the circuit is $(3-2)$ volts. The total resistance is $8+6+2+1$ ohms; hence $c = \frac{1}{17}$ amp.

Denote the potentials at X and Y by the small letters x, y . Take the connection from X to Y through AB and apply the formula noting that the direction $XABY$ is the same as that of the current and opposite to that of the E.M.F. of the cell AB . Hence

$$x - y - 2 = +\frac{1}{17}(4 + 2 + 3),$$

$$\therefore x - y = 2\frac{9}{17}.$$

Had we passed from X to Y through $A'B'$, we should have had

$$x - y - 3 = -\frac{1}{17}(4 + 1 + 3), \text{ i.e. } x - y = 2\frac{9}{17}.$$

85. Kirchhoff's Laws. I. If there is any network of conductors in which the currents have reached a steady state, then the total current entering any point is equal to the total current leaving that point.

This is evident, for if otherwise there would be an accumulation of electricity at the point, so that the distribution could not be steady.

(In Fig. 56, $c_1 + c_2 = c_3 + c_4$)

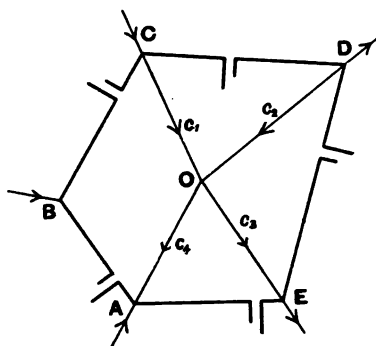


Fig. 56.

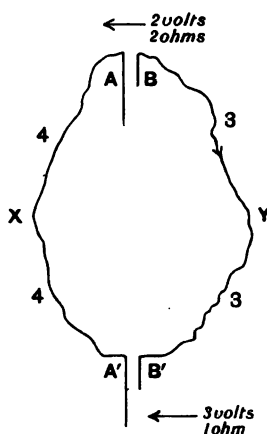


Fig. 55.

II. Suppose that in any network of conductors, such as Fig. 56, we take any closed circuit $ABCDEF$. Each arm will have a resistance: it may or may not include a cell or other source of E.M.F. Denote the current in the arms AB , BC , ... by the symbols $\mathbf{C}_{ab}$, $\mathbf{C}_{bc}$, ..., the electromotive force by $\mathbf{E}_{ab}$, $\mathbf{E}_{bc}$, ..., and the resistance by $\mathbf{R}_{ab}$, $\mathbf{R}_{bc}$, ..., the potentials at the points A , B , ... by a , b , Then by Ohm's Law

$$a - b + \mathbf{E}_{ab} = \mathbf{C}_{ab} \mathbf{R}_{ab},$$

$$b - c + \mathbf{E}_{bc} = \mathbf{C}_{bc} \mathbf{R}_{bc},$$

etc.

$$e - a + \mathbf{E}_{ea} = \mathbf{C}_{ea} \mathbf{R}_{ea},$$

$\therefore$ by addition $\mathbf{E}_{ab} + \mathbf{E}_{bc} + \dots \mathbf{E}_{ea} = \mathbf{C}_{ab} \mathbf{R}_{ab} + \dots \mathbf{C}_{ea} \mathbf{R}_{ea}$,

or

$$\Sigma \mathbf{E} = \Sigma \mathbf{C} \mathbf{R}.$$

This result is known as Kirchhoff's Second Law.

If in any closed circuit of a network we multiply the current in each arm by the resistance, the sum of the products thus formed is equal to the sum of the E.M.F.s of the circuit.

Examples. 1. Apply to the case of Wheatstone's bridge, Fig. 45, Art. 73,

(1) for the circuit $ABGD$, $0 = c_1 \mathbf{R} + 0 + (-c_2) \mathbf{K}$, i.e. $c_1 \mathbf{R} = c_2 \mathbf{K}$,

(2) for the circuit $BCDG$, $0 = c_1 \mathbf{S} + (-c_2) \mathbf{K} + 0$, i.e. $c_1 \mathbf{S} = c_2 \mathbf{K}$,

whence

$$\frac{\mathbf{R}}{\mathbf{S}} = \frac{\mathbf{K}}{\mathbf{K}}.$$

2. Apply to the potentiometer circuit, Fig. 51, Art. 80.

If i is the current along the potentiometer wire, $\mathbf{R}$ the resistance of AX , $\mathbf{E}$ the E.M.F. of the cell, then applying the second law to the closed circuit through AX , the galvanometer and the cell we get

$$i\mathbf{R} + 0 - \mathbf{E} = 0,$$

i.e. the E.M.F. of the cell is proportional to the resistance intercepted on the potentiometer wire.

3. Two cells are connected up as shewn in the diagram through resistances to the terminals of a galvanometer; find the current through the galvanometer.

Adopting the usual notation and applying the laws to the circuit through the galvanometer and the first cell

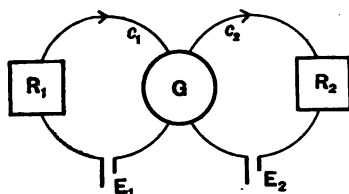


Fig. 57.

$$\mathbf{E}_1 = c_1 (\mathbf{B}_1 + \mathbf{R}_1) + (c_1 - c_2) \mathbf{G}.$$

Similarly

$$\mathbf{E}_2 = c_2(\mathbf{B}_2 + \mathbf{R}_2) + (c_2 - c_1)\mathbf{G}.$$

From these equations c_1 and c_2 may be determined.

A particular case is useful. Let $\mathbf{R}_1$ and $\mathbf{R}_2$ be so chosen that no current passes through the galvanometer, then $c_1 - c_2 = 0$ and the equations become

$$\mathbf{E}_1 = c_1(\mathbf{B}_1 + \mathbf{R}_1), \quad \mathbf{E}_2 = c_2(\mathbf{B}_2 + \mathbf{R}_2),$$

whence $\frac{\mathbf{E}_1}{\mathbf{E}_2} = \frac{\mathbf{B}_1 + \mathbf{R}_1}{\mathbf{R}_2 + \mathbf{B}_2} = \frac{\mathbf{R}_1}{\mathbf{R}_2}$ approximately if the external resistances are large compared with the internal.

EXAMPLES

1. Devise and sketch a B.A. bridge in which the wire is a metre long but is divided into two halves, placed parallel to one another and joined at one pair of ends by a strip of brass.

2. How could you readily test whether the wire of a B.A. bridge is of uniform resistance or not?

3. In a B.A. bridge the wire has a resistance of 0.5 ohm: the resistances to be compared are both much greater. Would it be of any service to replace the wire with one of higher resistance? Give reasons.

4. On a B.A. bridge the wire is one metre long. The unknown resistance is compared with an 8 ohm coil. What will be the position of the jockey on the wire in the following cases: (a) $x = 1.5$ ohms; (b) $x = 8$ ohms; (c) $x = 50$ ohms? If the experimenter makes an error of 3 mm. in the position of the jockey, by how much per cent. would this affect the calculated result in each case?

5. In the B.A. bridge, why is it desirable that the known resistance should be chosen of something like the same magnitude as the unknown?

6. Two cells are joined in series. Their common terminal is connected to a wire of resistance 3 ohms, the other end of which is connected by wires of resistance 2 ohms and 1 ohm to the remaining terminals of the cells. If the cells are each of E.M.F. 2 volts and resistance 1 ohm, find the current in each wire.

7. Why cannot the Carey Foster bridge be used unless (a) X and Y , (b) H and K are nearly equal?

8. In a Wheatstone Bridge arrangement the resistances in ohms of the arms are $R=3$, $S=2$, $K=5$. What must be the unknown resistance, X , if the conjugate condition is satisfied? If X has actually only half this value, find the P.D. between the galvanometer terminals when a total current of 1 ampere is being taken from the battery.

w.

6

9. What is the fall in potential per centimetre along a potentiometer wire, 8 metres long and resistance 26 ohms, when a current of 0.4 ampere passes along it?

10. When a Clark cell is on a potentiometer the jockey is 259 cm. from the end. Where will it be for a cell of E.M.F. 1.08 volts? If the current in the potentiometer was 0.28 ampere while the Clark was under test, but fell to 0.24 while the second cell was being tested, what would be the position found? What error would be made in calculating the E.M.F.?

11. $AXBY$ is a square formed of 4 wires XA , AY , YB , BX of resistances 1, 2, 3, 4 ohms respectively. At X and Y cells of E.M.F. 1.2, 1.1 volts, internal resistances 0.8, 0.9 ohm respectively are introduced. If the cells be placed in opposition, find the P.D. between A and B .

12. The positive poles A and B of a Grove and Daniell cell are joined by a wire of 0.3 ohm resistance, and the negative poles C and D by a wire of 0.5 ohm. What is the difference of potential between the middle points of AB and CD ?

Grove cell : resistance = 0.2 ohm, E.M.F. = 1.8 volts.

Daniell cell : resistance = 0.4 ohm, E.M.F. = 1.1 volts.

13. A circuit is made up of (1) a battery with terminals A , B , its resistance being 3 ohms, and its E.M.F. 2.7 volts; (2) a wire BC of resistance 1.5 ohms; (3) two wires in parallel circuit, CDF , CEF , with respective resistances 3 and 7 ohms; (4) a wire FA of resistance 1.5 ohms. The middle point of the last wire is put to earth: find the potential at the points A , B , C , F .

14. The terminals of a battery, of E.M.F. 4 volts and resistance $1\frac{1}{2}$ ohms, are connected to those of a battery, of E.M.F. 3 volts and resistance $\frac{1}{4}$ ohm, by wires of resistances 1 and 6 ohms respectively, so that both batteries act in the same direction. Shew that if a third wire be placed so as to join the middle points of these wires no current passes through it.

15. The current from a Daniell cell (E.M.F. 1.08) passes through a resistance of 1000 ohms and a potentiometer wire in series. If the wire is 10 metres long and has a resistance of 2.3 ohms per metre, find the fall in potential per metre along the wire.

16. Explain with a diagram how a potentiometer in conjunction with a set of standard resistances might be used to test the readings of an ammeter.

17. How might a potentiometer be made to measure a small potential difference, say 0.001 volt? What kind of wire should be used in a potentiometer? Should it be thick or thin; of high resistance or of low?

CHAPTER IX

GALVANOMETERS

86. The Electromagnetic System of Units. As in mechanics it is convenient to build up a system of units—dyne, erg, watt, etc.—which are simply related to one another and derived by definition from the centimetre, the gramme and the second; so in electricity and magnetism we find it well to define our units by reference to these three units of length, time, mass and one other. This fourth is the unit magnetic pole. It is defined in Art. 4. In Chapter I we saw that a compass needle was affected by a current: that a current has a magnetic field. Suppose a wire carrying a current were wrapped into a ring of one centimetre radius. A unit north pole placed at the centre of this would experience a force at right angles to the plane of the coil. If the number of dynes in this force is the same as the number of centimetres in the wire, the strength of the current is said to be one *electromagnetic unit*. In other words, if a current flowing round a ring of wire (one turn only) of radius one centimetre exerts a magnetic force of 2π dynes at the centre, the measure of the current is *one*, if we adopt the C.G.S. electromagnetic system.

Unit charge is the quantity of electricity conveyed in one second by a unit current.

The unit of E.M.F. between two places is that which exists if unit work (one erg) is done by the passage of unit charge from one to the other. (*v.* also p. 118.)

The unit resistance is that of a connection in which unit current flows under unit potential difference.

87. Practical Units. These C.G.S. electromagnetic units however, though useful for scientific work and measurements, would be cumbersome in everyday use. More convenient ones have therefore been devised, (a) the ampere, Art. 26, (b) the volt, Art. 70, (c) the ohm, Art. 48.

The definitions of these quantities are quite arbitrary but they were chosen so that as nearly as possible

1 ampere = 10^{-1} C.G.S. unit current,

1 volt = 10^8 C.G.S. units of electromotive force,

1 ohm = 10^9 C.G.S. units of resistance.

The only other practical unit which we need mention is the *farad*: it is the unit of capacity. A condenser has a capacity of a farad if its charge is a coulomb when the P.D. between its plates is a volt. It is a very big unit and its millionth part, the *microfarad*, is in general use.

88. Force due to current in a circular coil. The force exerted by a current on a unit pole would be very hard to measure directly, but it is easy to compare that force with the earth's magnetic field. To do this suppose that a coil of wire of any radius is so placed that when a current passes round it the force exerted at the centre of the coil is horizontal and due East or West. We can arrange this if the plane of the coil is vertical and its axis East and West: then its plane is in the plane of the magnetic meridian. Call the force due to the current **F**. Call the horizontal component of the earth's field **H**. Then the horizontal forces acting on a unit pole at the centre of the coil are **F**, due East or West, **H** due North.

By the parallelogram of forces, we know that the resultant of these two forces will be

inclined to **H** at an angle δ such that $\tan \delta = \frac{\mathbf{F}}{\mathbf{H}}$ (v. Art. 8).

Now a small compass needle at the centre of the coil would point in the direction of the resultant force there: i.e. would be deflected from the meridian through the angle δ , where $\mathbf{F} = \mathbf{H} \tan \delta$.

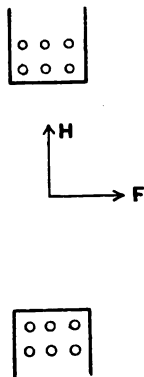


Fig. 58.

89. To find how the force due to a current in a galvanometer coil depends on (1) the strength of the current, (2) the number of turns of wire, (3) the radius of the ring.

(1) Put in series with the galvanometer an ammeter and a variable resistance and join up to the terminals of a cell. Find the reading of the ammeter and galvanometer with different resistances in the circuit.

Enter the results in columns headed (1) Current, (2) Deflexion, (3) Force due to current ($= H \tan \delta$), (4) Current : force. Plot the relation between the current and $\tan \delta$ on squared paper.

(2) To find the effect of the number of turns it is convenient to use an unwound galvanometer, a cell, and a long wire.

Connect the two terminals of the cell by the wire. Place the galvanometer with the coil in the plane of the magnetic meridian. Now wrap a single coil of wire on the ring and note the deflexion. Repeat with two, three, four, etc., coils of wire.

Put your results in tabular form shewing the relation between the number of coils and the tangent of the angle of deflexion.

(3) To find the effect of the radius of the ring we make use of a galvanometer with concentric windings of different radii. Pass a current through one winding: notice the deflexion and the current. Calculate the force per turn per ampere. Repeat with the other windings. Tabulate your results to shew how the force per turn per ampere changes with the radius.

We could vary the method by using an unwound galvanometer with two rings and a long wire. Connect the terminals of the cell with a long wire. Wrap any number of coils (say 2) of this wire round the inner ring. Now lead the wire to the outer ring and wind the wire in the reverse direction, putting on just so many coils as will suffice to bring back the needle to the zero position. Compare the numbers of the coils with the radii. Repeat the experiment varying the number of coils on the first ring.

90. Reduction Factor. These experiments tell us that if a current flows round a galvanometer coil, the force that it produces at the centre is (1) proportional to the strength, (2) proportional to the number of turns, (3) inversely proportional to the radius.

Now a current of 1 ampere flowing round a ring of one centimetre radius exerts a force of $\frac{2\pi}{10}$ dynes at the centre; hence C amperes will exert a force of $\frac{2\pi nC}{10 r}$ dynes at the centre of a ring of n turns and radius r cm.

If this force, F , produces a deflexion δ of a small magnet at its centre, we know that

$$F = H \tan \delta = \frac{2\pi nC}{10r}, \quad \text{i.e. } C = \frac{10Hr}{2\pi n} \tan \delta.$$

The fraction $\frac{10Hr}{2\pi n}$ is called the reduction factor of the galvanometer. It is usually denoted by K .

The expression $\frac{2\pi n}{r}$ is spoken of as the galvanometer *constant*.

In many galvanometers it is not easy to measure either n or r . Hence the reduction factor must be determined by experiment. To do this connect up in series with an accurately graduated ammeter and find the current. Note the current and the deflexion. Substituting in the equation $C = K \tan \delta$, we find the value of K . Once K has been found, we can measure any current without difficulty by a tangent galvanometer. As a rule however we do not use galvanometers to measure currents but only to compare one current with another. (See also Arts. 30, 31.)

91. Reverser. In using a galvanometer for accurate work it is often connected up through a reverser. This helps to eliminate any error due to the galvanometer not being set up exactly in the meridian.

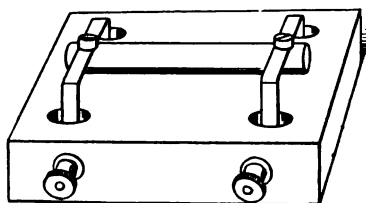


Fig. 59.

The reverser shewn in the figure consists of two parts. The one is a block of ebonite in which are drilled four cups, half filled with mercury and connected to the four terminals at the sides. The other part is a frame of two parallel copper bars, the ends turned down to dip into the cups. The bars are joined by a piece of ebonite. A pair of cups diagonally opposite are joined to the galvanometer. To reverse the current lift the frame and turn through a right angle before replacing.

92. Sine Galvanometer. The plane of the coil of a *tangent* galvanometer must always be in the meridian: the current is proportional to the tangent of the angle between the needle and this plane. In the *Sine* galvanometer the coil is turned about a vertical diameter until its plane contains the axis of the deflected needle. The sine of the angle which it makes with the meridian is proportional to the current.

This is sufficiently explained by fig. 60.

Here $\mathbf{F}$ as before is equal to $\frac{2\pi n\mathbf{C}}{10r}$ and

is at right angles to the plane of the coil but not to the meridian. The axis of the needle is in the direction of the resultant of $\mathbf{F}$ and $\mathbf{H}$: therefore $\mathbf{F} = \mathbf{H} \sin \delta$,

i.e. $\mathbf{C} = \frac{10\mathbf{H}r}{2\pi n} \sin \delta$.

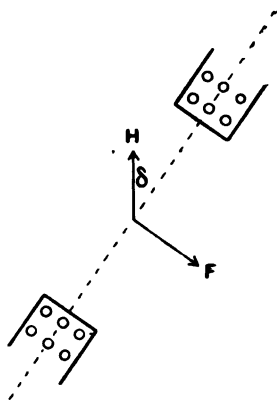


Fig. 60.

The instrument is seldom used and is of little importance.

93. Kelvin's Reflecting Galvanometer. In this the external field is due partly to a magnet *NS* supported over the needle *ns* (figs. 61, 62). The needle itself is a magnetised

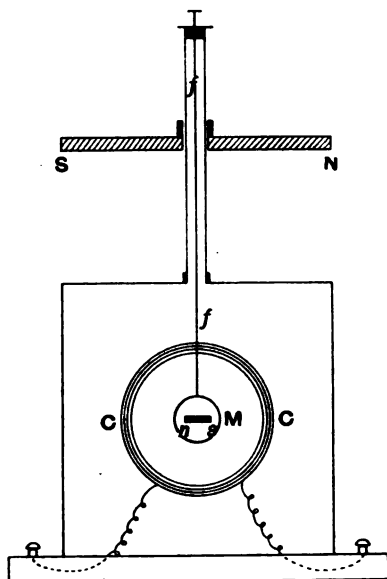


Fig. 61.

piece of watch-spring stuck to the back of a light mirror M . The combination is suspended at the centre of the coil, CC , by a fibre, f , free from torsion. In place of a pointer fixed to the needle, a beam of light thrown from a lamp and reflected by the mirror is focussed by a lens on a scale.

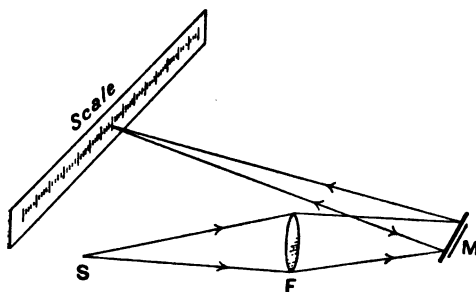


Fig. 62.

As usually arranged the lamp is used to illuminate a narrow vertical slit, S . The light diverging from this is made to converge by the lens F , then reflected by the mirror M and finally focussed on a graduated horizontal scale fixed above the slit. When no current passes, the image of the slit should fall on the zero of the scale.

If the mirror is deflected through an angle θ , the reflected beam is deflected through an angle 2θ (for the normal to the mirror turns through θ and the angle between the incident and reflected rays is 2θ). If then d is the distance from scale to mirror, s the distance of the image from the zero, $s/d = \tan 2\theta$.

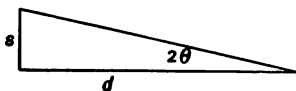


Fig. 63.

If the angle θ is small, $\tan 2\theta$ is practically equal to 2θ or to $2 \tan \theta$. Hence $\frac{s}{d} = 2 \tan \theta = 2 \frac{C}{K}$, where C is the current and K the reduction factor. This result shews that for small angles the current is proportional to the scale deflexion.

In some cases the reduction factor K may be calculated as in Art. 90, if we know the strength of the field; it would, however, generally be much simpler to find it experimentally, by passing a small known current through the instrument and

noticing the deflexion. The constants are generally stated by the makers. For comparison of currents a knowledge of its value is not necessary. If the mirror M is concave, the lens F is not necessary.

A word must be added about the control magnet. This is adjustable in two ways, (a) it can be twisted about the vertical, (b) it can be raised or lowered. The first adjustment enables the image of the slit to be brought to the zero of the scale. The second alters the sensitiveness. If the magnet is low the field is strong; if it is raised the field is weakened. Usually the galvanometer is placed in such a position that the mirror is due east of the lamp, so that if the control magnet is sufficiently raised the field (due to earth and control) in which this mirror magnet lies may be brought to almost any desired strength. If the field of the magnet exactly neutralised the field of the earth, then the position of the mirror would be governed only by the torsion of the support and a very small current would produce a large deflexion.

94. In "Moving Coil" galvanometers the external field is due to a strong permanent magnet NS between the poles of which hangs the galvanometer coil cc , suspended by a light torsion wire ω through which the current enters. The position of the coil is accurately indicated by light reflected from a mirror M . A cylinder of iron a is fixed between the poles. This is only just cleared by the coil. It serves to concentrate the field (*v. fig. 133*). When no current is passing the coil should hang evenly between the magnet poles with the normal to its plane at right angles to the lines of magnetic force. The deflexion produced is nearly proportional to the magnitude of the current.

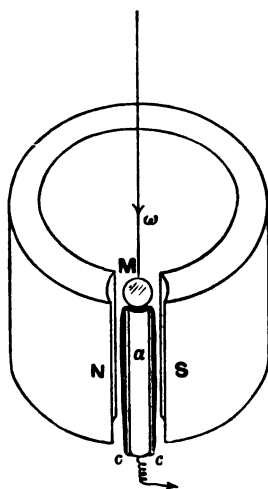


Fig. 64.

95. Ammeters. An ammeter differs from a galvanometer in that it measures a current directly in amperes.

There are many different kinds in use. One of the best consists of the following parts:

(1) A strong permanent magnet, to which iron pole pieces (P) are fixed, producing a field independent of the earth.

(2) A moving coil cc through which the current passes.

(3) A pair of springs like the hair spring of a watch, which regulate the motion of the coil, and lead the current to and from it.

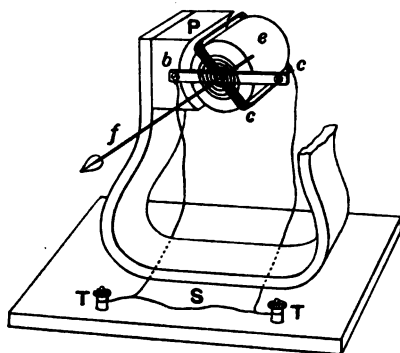


Fig. 65.

In the figure, cc is the moving coil. For convenience of description part of the magnet is supposed to be broken off. The front spring is shewn fastened at the centre to the bar b ; the other, coiled the opposite way, is at the back.

The coil of wire is so shaped that it can move in the cylindrical space which separates the pole pieces of the magnet. It is mounted with its axis along the centre of this space. When no current passes, the springs bring the plane of the coil at 45° to the lines of force between the poles. When a current is flowing, the forces which it calls into existence tend to twist the coil round in the direction $bceP$ until its plane is at right angles to the lines of force; the magnitude of these forces is dependent on the current. The angle through which the coil turns will therefore be dependent on the relative values of the strength of the springs and of the current. This angle is measured by a long light pointer f fixed to the coil which moves over a scale marked in amperes. The current is brought from the terminals T, T of the in-

strument to the coil by means of the governing springs. As in the case of the moving coil galvanometer a soft iron cylinder c is placed at the centre of the coil. The pivots of the coil have their bearings in this.

Now it is evident that a coil like this will not stand large currents: a current of an ampere would burn it out; so that if it is required to measure such currents it must be shunted. The shunt is shewn at S in the figure. There is very little difference then between an instrument intended to measure an ampere and one intended to measure 200 amperes. The same mechanism will suffice for the two, but the one intended for large currents must be shunted with a lower resistance.

96. Voltmeters. The ordinary voltmeter is similar in construction but differs in that the coil instead of being shunted is placed in series with a high resistance. The instrument, therefore, tells the voltage between two places by measuring the current which passes between them when joined up through a definite high resistance. It is useless for measurements in static electricity for, obviously, it only measures *current*.

97. Astatic Galvanometer. If two magnetised needles exactly similar were fixed parallel to one another with unlike poles together, then it is evident that the action of the earth's field on one needle would be neutralised by the action of the other: the combination would not tend to point in any definite direction. Such an arrangement is called Astatic.

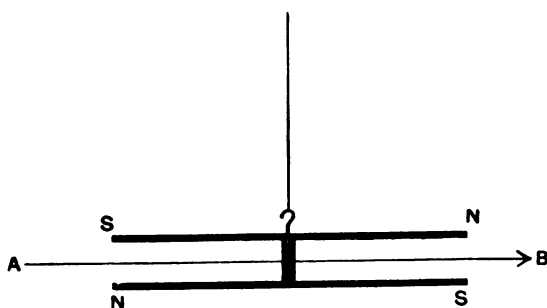


Fig. 66.

Now suppose an astatic combination is suspended as in the figure and that a current flows along the wire AB . The

effect of this current will be to tend to make *both* needles turn in the same sense at right angles to the wire: the only force to hinder this is that exerted by the torsion of the support.

The effect of the current may be increased if a coil of wire is wrapped round one of the needles: if a coil surrounds each, the windings must be in opposite directions (fig. 67).

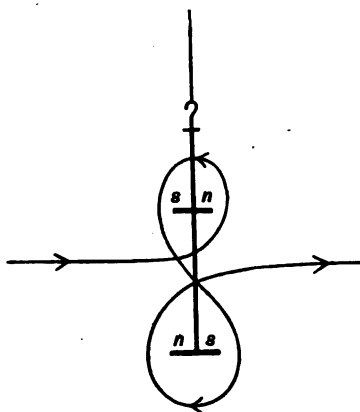


Fig. 67.

A combination of magnets fixed with opposite poles together would be perfectly astatic if (1) the axes were parallel, (2) the magnetic moments were equal. These conditions cannot be realised in practice, so that the position of the magnets is in reality controlled to some extent by the earth's field. Astatic galvanometers are used more for detecting small currents than for measuring them.

98. A Ballistic Galvanometer is used to measure a quantity of electricity: not the strength of a current. Its construction however is similar to an ordinary galvanometer in that it consists of a magnetic combination and a coil. We shall suppose the coil fixed and the magnet free to move. If a battery is suddenly connected up through a ballistic galvanometer to the plates of a condenser, a large current will flow for a short time. The total charge which passes is measured by the product of this time and the average current. In passing through the coil, the current exerts a force on the

magnet and so gives it an impulse or blow; the effect of which may be compared with the effect of a horizontal blow on a pendulum bob. The deflexion (θ) produced and the total charge (Q) are connected by the relation

$$Q = \frac{kT}{\pi} \sin \frac{\theta}{2},$$

where k is the reduction factor of the galvanometer and T is the time of free oscillation.

Usually a reflecting system is adopted and the deflexion θ is small: in this case the charge is nearly proportional to the throw.

EXAMPLES

1. Two cells are connected up in turn with a high resistance galvanometer and give deflexions of 23° and 13° . If the E.M.F. of the first is 1.2 volts, what is that of the second?

2. A coil of six turns, each of which is 0.2 metre in diameter, deflects a compass needle at its centre through 40° . Find the strength of the current. ($H=0.18$.)

3. A galvanometer (resistance 2 ohms) is placed in circuit with a cell of E.M.F. 1 volt and resistance $\frac{1}{2}$ ohm. If the coil has 20 turns of diameter 10 cms., what deflexion will be produced? ($H=0.18$.)

4. A battery when joined to a tangent galvanometer of 10 ohms resistance gives a deflexion of 60° . If a resistance of 20 ohms is inserted in the circuit the deflexion falls to 45° . What is the resistance of the battery?

5. The coil of a tangent galvanometer is placed at right angles to the meridian. Would it be possible to determine the direction or magnitude of a current which can be made to flow through it?

6. Explain the advantages and disadvantages of a suspended coil galvanometer as compared with an instrument with a suspended magnetic needle.

7. A tangent galvanometer has two concentric windings, one of radius 5 cm. and 18 turns, and the other of radius 15 cm. and 30 turns. What deflexion will be produced by a current of 0.4 ampere which flows round both in the same direction? ($H=0.20$.)

8. A battery of 6 cells in series, an ammeter and a key are arranged in circuit. The poles of the battery are also connected to the terminals of a high resistance voltmeter. The voltmeter registers 12.4 volts *before*, and 10.6 volts *after*, the ammeter indicating 15 amperes. What is the resistance of each cell of the battery?

9. In a tangent galvanometer is there any need for the coil to be circular?

10. If the needle of a tangent galvanometer is not exactly in the middle of the card, will the readings be correct?

11. Has the sine galvanometer any advantage over the tangent galvanometer?

12. In a mirror galvanometer the lamp and scale are one metre away from the mirror. If a current of .0006 ampere moves the spot of light 3.2 cm. on the scale, what is the reduction factor of the galvanometer?

13. In the moving coil galvanometer is the position relative to the meridian of any importance?

14. How is the magnetic combination in a so-called astatic galvanometer controlled? On what does its sensitiveness depend? Can it be used for measuring currents?

15. In some forms of ammeters the moving system is controlled not by a spring but by gravity. Discuss the merits of the gravity control.

16. A tangent galvanometer is connected in series with a copper voltmeter and a source of current and gives an average deflexion of 45° during the 20 minutes that the current is passing. It is found that 0.4 grm. of copper are deposited on the kathode. Calculate the reduction factor of the galvanometer.

17. Shew that in a tangent galvanometer the deflexion is independent of the moment of the magnet, and explain why it is well to have a magnet which is both strong and short.

18. On what conditions does the sensitiveness of a galvanometer depend?

19. Shew that in a Kelvin reflecting galvanometer the current is nearly proportional to the distance of the spot of light from the zero.

20. A galvanometer, the resistance of which is $\frac{1}{2}$ ohm, being joined up in circuit with a cell by thick copper wires, the resulting current is noted; and it is found that the current in the galvanometer is halved if, without any other change being made, the terminals of the galvanometer are joined by a wire of resistance 0.1 ohm. What is the resistance of the cell?

21. Shew that if a slight error is made in reading the angle of deflexion of a tangent galvanometer, the percentage error in the reduced value of the current is a minimum if the angle of deflexion is $\pi/4$.

22. A galvanometer, resistance R , is joined up to a cell by wires of negligible resistance. If the galvanometer is shunted by a wire of resistance x , shew that half of the original current flows through the galvanometer if the internal resistance is $Rx/(R-x)$.

23. In a tangent galvanometer the pointer is bent so that it registers in every case a deflexion too great by an angle α . It is used to compare the E.M.F. of two cells. Prove that the ratio of the true E.M.F. to the value found from this galvanometer is approximately

$$\tan \beta / (2 \tan \alpha + \tan \beta),$$

where β is the sum of the deflexions given on a correct galvanometer, and α is small.

24. A galvanometer of 200 ohms resistance is in series with a resistance of 300 ohms and a battery which has an E.M.F. of 3.4 volts and an internal resistance of 12 ohms. Find the current passing through the galvanometer. Also find the value of the current (i) when a shunt of 20 ohms is placed across the galvanometer, and (ii) when the same shunt is placed across the battery.

CHAPTER X

TEMPERATURE EFFECTS

99. Temperature and Resistance. To find the effect of temperature on the resistance of a wire.

Take a couple of yards of fine copper wire, say number 28, coil it up and join its ends to thick leads by means of binding screws and immerse it in a water bath. The leads are to be joined to the resistance gap in a B.A. bridge or P.O. box. Raise up the temperature of the water to 100°C . and find as in Art. 74 or 75 the resistance of the wire. Allow the bath to cool, but keep it well stirred. Take the readings of the resistance at different temperatures between 0°C . and 100°C . Tabulate the results and plot a curve between resistance and temperature.

In all metals the resistance increases with rise in temperature. With pure copper, platinum, silver, gold and some others an approximate law connecting R_t the resistance at $t^{\circ}\text{C}$. and R_0 the resistance at 0°C . is

$$R_t = (1 + \alpha t) R_0,$$

where α is different for the different metals but approximately equal to '0038.

A comparison of this with the gas pressure law of Gay-Lussac, $P_t = (1 + '00366t) P_0$, seems to indicate the existence of an absolute zero, near to -273°C ., at which these pure metals are perfect conductors.

In carbon and electrolytic solutions the resistance decreases with the temperature.

100. Seebeck Effect. If two wires of different materials, say copper and iron, are joined up to form a single circuit, and if one of the junctions is heated, a current of electricity

will circulate. An easy way to shew this is to join one end of each wire to the terminals of a sensitive galvanometer, twist the free ends together and warm them. The warmth of the fingers is often sufficient to produce a sensible deflexion.

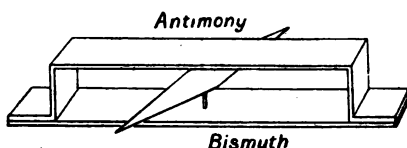


Fig. 68.

Another simple method of shewing the existence of thermo-electric currents is to heat one junction of a bismuth-antimony couple (fig. 68). If the frame is held in the meridian the compass needle mounted in the centre will be deflected.

101. E.M.F. and Temperature. To find how the E.M.F. of a copper-iron junction varies with the temperature. (See also qu. 29, p. 242.) If the range of temperature is from 0°C. to 100°C. the arrangement in the diagram (fig. 69) is convenient.

An accumulator *E* or other cell of constant E.M.F. is joined up to a resistance coil *R* and a potentiometer wire *AB*.

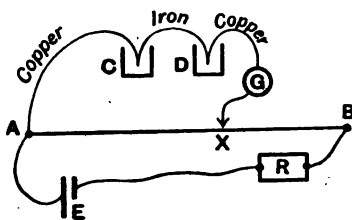


Fig. 69.

The junctions of the couple are placed in beakers *C*, *D*, one containing water kept at 0°C. by lumps of ice, the other containing water at any temperature required. The couple is made of bare iron wire soldered or joined by binding screws to the copper wires.

Begin with boiling water in the hot bath. Find as in the experiment, Art. 81, the point *X* at which contact must be made with the potentiometer wire for no current to pass through the galvanometer *G*. Measure the distance *AX*.

Allow the hot bath to cool and take readings for about every ten degrees fall. Plot a curve between the difference in the temperature of the baths and the lengths along the potentiometer wire. This shews the relation between the E.M.F. and the temperature.

The resistance coil R must be properly chosen: if it happens that the first point X comes quite close to A , R is too small. If it is impossible to find X at all, either R is too big or the cell is coupled up the wrong way. No definite directions as to choice of R can be given. It depends on the length and the resistance of the potentiometer wire. Try first with $R=2000$ ohms and change if necessary.

To find the actual value of the E.M.F. due to the thermo-couple we must know the resistance of R and of the potentiometer wire per centimetre as well as the E.M.F. of the cell.

Suppose R is 2000 ohms, the E.M.F. of the cell 1·08 volts, the length of the wire 6 metres and its resistance per metre 1·23 ohms, and that AX is 285 cm. long. Now the fall in potential between the poles of the cell on being connected up with a resistance of over 1000 ohms is very small and may be neglected. Hence the current along the potentiometer wire is $\frac{1\cdot08}{2000 + 6 \times 1\cdot23}$ amperes, i.e. ·000538 amp. The fall in potential per centimetre along the wire is therefore $\cdot000538 \times \cdot0123$ volt.

The E.M.F. of the thermo-couple is then $\cdot000538 \times \cdot0123 \times 285$ volts, i.e. ·00183 volt.

The E.M.F. obtainable from a copper-iron couple is very small, rather less than 20 microvolts per degree centigrade. A bismuth-antimony couple has five or six times this value. It is used in thermopiles for detecting radiant heat. In this instrument many couples are joined zigzag in series and the alternate junctions exposed to the source of heat.



Fig. 70.

102. Neutral Temperature. The graph obtained by the experiment in Art. 101 seems to indicate that the E.M.F. of the thermo-couple is nearly proportional to the difference in temperature: this is true under the conditions. If, however, the temperature of the hot junction is gradually raised, the current, instead of flowing round in the direction of the diagram, fig. 71, and increasing, begins to diminish after a certain temperature 270°C. is reached, vanishes at 540°C. ($2 \times 270^{\circ}$) and then reverses. The temperature 270°C. at which the current begins to diminish is independent of the temperature of the other junction: it is also the mean of any two different temperatures at which the junctions may be maintained to give no current and is therefore called the neutral temperature.

103. Peltier Effect. The existence of thermo-electric currents was discovered in 1821 by Seebeck and the phenomenon is called after him—the Seebeck effect. The converse of this is known as the Peltier effect. If a current flows across a junction of two different metals, then heat is either absorbed or liberated at the junction, i.e. the junction is either cooled or warmed. If, for instance, a battery drives a current round a copper-iron circuit in the direction shewn in fig. 72, then the junction at which the current flows from the iron to the copper is warmed and the other is cooled.

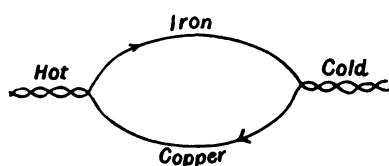


Fig. 71. Seebeck effect.

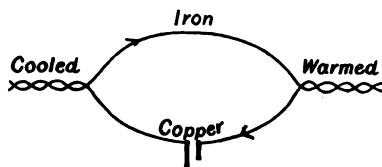


Fig. 72. Peltier effect.

We know that a current passing through any conductor warms it: this effect, described in Art. 54, and known as the Joule effect, is generally sufficient to mask the Peltier effect so that it is not very easy to illustrate the latter: it may be done, however, by using a bismuth-antimony couple and placing the junctions in the bulbs of a differential air thermometer.

104. Measurement of temperature. The mercury-in-glass thermometer has a very limited range, for mercury boils at 350°C . and freezes at -40°C . To measure temperatures outside this range some other form of instrument is necessary. A gas thermometer is useful at low temperatures, but not very convenient; at high temperatures there is difficulty in getting a container which neither melts nor becomes porous. We have therefore to use electrical thermometers. The two most common types are (a) the resistance thermometer, (b) the thermo-electric junction.

105. Resistance thermometers. Figure 73 shews a thermometer which consists of a coil of fine platinum wire wound on a frame of mica. This resistance is the essential part of the instrument. It is joined up by thick leads to the

binding screws shewn and protected by a case or tube. If really high temperatures are to be measured, the leads must be of platinum and the tube of porcelain: otherwise copper and steel will suffice.

When the instrument is in use the end is placed in the furnace or other place of which the temperature is required and the terminals joined up by wires to the "unknown" gap on a B.A. bridge.

The resistance is then determined and the temperature calculated.

Prof. Callendar and others have made accurate investigations on the resistance of platinum at the temperatures measured by the air thermometer. For our purpose it is sufficient to say that between 0° C. and 1000° C.

$$r = at + \frac{bt}{100} - \frac{bt^2}{10,000},$$

where r = increase in resistance (measured in ohms) above resistance at 0° C.,

t = temperature centigrade,

a = increase in resistance per degree between 0° C. and 100° C.,

b = a number dependent on the condition of the platinum used and equal to about 1.5.

In practical use it is often quite unnecessary to know the actual temperature on the air scale: it is sufficient to know it on the platinum scale: in other words the temperature is considered to be proportional to the excess in resistance.

In figure 73 are shewn two pairs of leads and terminals. These are necessary, for if the thermometer is placed in a furnace not only is the resistance wire heated but also the connecting leads: the resistance of the latter rises, so that the total increase depends on what length of the stem is heated. To compensate for the uncertainty thus introduced a pair of dummy leads—shewn by a single line down the centre—is inserted. Being exactly similar to the real leads they suffer the same increase in resistance: this can be ascertained or eliminated.

106. Thermo-electric thermometers. One of these is illustrated in figure 74. It needs little description. The wires shewn in the interior of the tube are of different metals

or alloys: perhaps of iron and nickel; or of platinum and platinum-iridium. They are joined to the terminals at the top and welded together at the bottom. The platinum form may be used up to 1500°C . and gives consistent readings. The temperature may be assumed to be roughly proportional to the E.M.F. developed. The latter is read by a high resistance millivoltmeter.

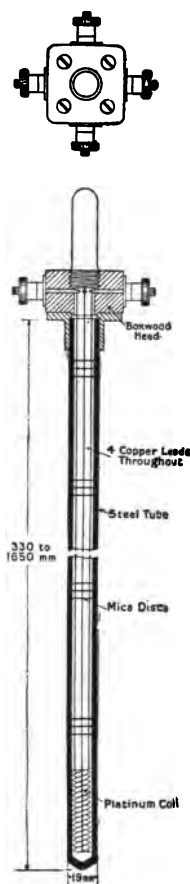


Fig. 73.

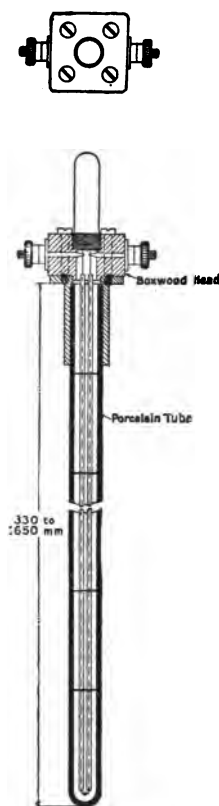


Fig. 74.

EXAMPLES

1. The temperature coefficient of platinum is $\cdot 0032$. The resistance of a length of wire of this substance is 4 ohms at 0°C . With what resistance must this be shunted at 50°C . to reduce the resistance at that temperature to 4 ohms?

2. A length of manganin wire has a resistance of 187.5 ohms at 0°C .; the temperature of the wire is 17°C . What is the error in the resistance if the temperature coefficient of the wire ($\cdot 000025$) is not taken into account?

3. A thermopile is joined up in series with a Daniell cell, and the current allowed to flow for a short time. The thermopile is then removed from the circuit and connected to the terminals of a galvanometer, the needle of which is thereupon considerably deflected, but gradually returns to its undisturbed position. Explain this.

4. Assuming the thermo-electric powers of iron and nickel with respect to lead to be +12 and -20 microvolts respectively, find the E. M. F. of a nickel-iron couple with junctions at 0° and 100°C .

5. A ring is made, partly of iron and partly of copper wire, the junctions being *A* and *B*. If *A* be kept at 0° and *B* at 100° , a thermo-electric current is produced in the circuit. Similarly a current is produced if *A* be at 100° and *B* at 200° . Have the currents in each case the same strength? Give reasons for your answer.

6. Under what circumstances can an electric current cool the conductor through which it passes? What will be the result if the direction of the current be reversed?

7. The resistance of a copper wire is determined by a Wheatstone's Bridge box when the temperature of the air is 20°C . and is found to have the value 20.25 ohms. Calculate its true value at 0°C ., the coils of the box being of German silver and correct at 15°C . Assume temperature coefficients of copper and German silver to be $\cdot 004$ and $\cdot 002$.

8. In a B. A. bridge the ratio arms are equal. The other two resistances *K*, *X* are connected up by exactly similar leads. By change of temperature the resistances of *X* and the leads to *K* and *X* are increased. The increase in the resistance of *X* is calculated by measurements on the bridge. Will this calculation need to take into account the alteration in the resistance of the leads?

CHAPTER XI

MAGNETIC ACTION OF A CURRENT

107. WE have seen in Chapter I that a magnet exerts forces on conductors carrying currents and that the currents exert a reaction on the magnet. We are now going to investigate these actions a little more closely.

Suppose we take a ring of wire in which a current is passing. In the figure the ring is supposed to be nearly at right angles to the plane of the paper and the current flows down the near and up the far side, so that, seen from the magnet NS , it goes round the circuit in the same way as the hands of a watch. The magnet is held at right angles to the

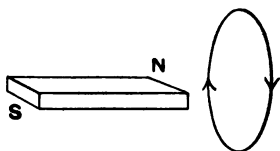


Fig. 75.

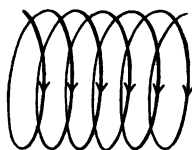


Fig. 76.

plane of the coil and so almost in the plane of the paper. Our cork-screw rule tells us that a free north pole anywhere on the axis of the coil would be pulled through the coil from the left-hand side to the right. Hence there will be attraction between the coil and magnet. This attraction can be very much increased if several turns of wire are used. The wire may be wound on a ring (cf. the galvanometer) or it may be wound in a helix on a tube in which case we get a *solenoid* (fig. 76).

108. Magnetic action of a Solenoid. The lines of force due to the current flowing in a galvanometer ring may be traced by the method indicated in Art. 11. They are found to be exactly similar to those of a bar magnet. The only difference is that in a solenoid the lines can be traced right through the interior; in a solid magnet this is obviously impossible. In these particulars a coil or helix of wire carrying a current seems to act exactly like a magnet. Let us see if we can produce some other magnetic effects.

A magnet attracts iron filings. Wrap a helix of wire on a paste-board tube, place some iron filings near one end of the tube and pass a strong current through the wire. The filings will be sucked up.

Magnets placed end to end repel one another if like poles are together; attract if unlike are together. Hang up side by side two coils of wire and arrange them to swing freely. The free ends of the wire should be left long for this purpose. Join them in series and pass a current in the same direction round each. They will attract one another. If the current in one is reversed, they will repel.

If we use one coil only it will be attracted or repelled (according to the direction of the current) by the pole of a magnet.

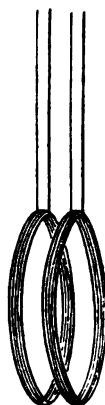


Fig. 77.

109. Parallel Currents. To shew the action between parallel currents, it is not necessary to wrap the wire into coils. If two flexible wires—tinsel answers very well—are hung up side by side about a $\frac{1}{4}$ inch apart and a current is passed down (or up) both of them they will move nearer together: but if the current passes up one and down the other they will separate. Hence with parallel currents in the same direction we get attraction; and with parallel currents in opposite directions we get repulsion.

110. Floating Cell. A magnet capable of rotating about a vertical axis points to the North in the Earth's field. De La Rive's floating cell shews the corresponding effect with a current in a coil of wire. A beaker containing dilute sulphuric acid floats in a pan of water. In the beaker

are immersed a plate of zinc and a plate of copper. These are joined together (see fig. 78) by a helix of copper wire. The current makes the helix turn round in the Earth's field till the axis points North and South.

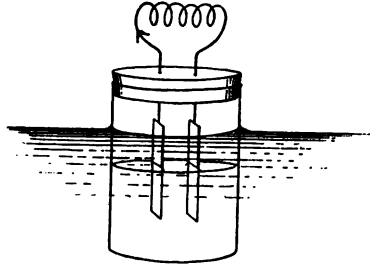


Fig. 78.

111. Electromagnet. A piece of soft iron or steel becomes magnetised by induction in the presence of a magnet. In the case of a helix we can place the iron right inside. In this way we get an electromagnet. Take a short rod of soft iron and wrap it with several turns of insulated wire. Test its magnetism by means of filings or a compass. The first electromagnet appears to have been made by Sturgeon in 1825.

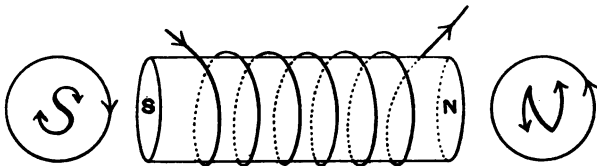


Fig. 79.

A horse-shoe magnet must be wound so that one end is a North pole and the other a South. Remember that if a current passes round a bar of iron in such a way as to appear clockwise to the observer, the end of the iron that points to him is South. See fig. 79.

112. The Electric bell. (Fig. 80.) *AB* is a soft iron plate: *MM* an electromagnet. The spring *S* keeps the platins *P, p* in contact when *MM* is not excited. The current enters at the terminal *T'*, passes round the magnet, through *S, P, p* and back by the terminal *T* to the battery. While the current passes the magnet attracts *AB* and pulls it up so that the hammer strikes the bell: the platins *P, p*

separate, the current is broken, the iron loses its magnetism and *AB* is brought back to its old position by the spring *S*. The current now starts again and the cycle is repeated. Every time the current is broken at *Pp*, there is a spark: hence the use of the platinum tips, for steel or brass would very quickly burn away.

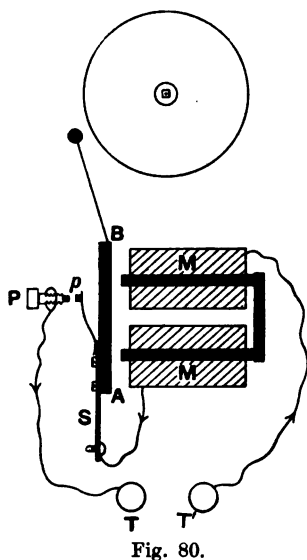


Fig. 80.

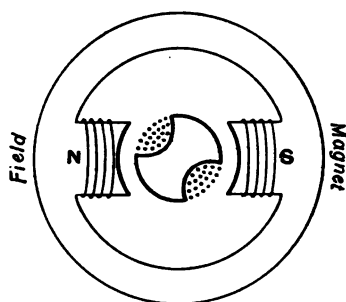


Fig. 81.

113. A simple electromotor may be made of the following parts.

The field magnet. A soft iron shell with the pole pieces *N* and *S* is cast. The pole pieces are wound with insulated wire, fig. 81.

The armature, fig. 82, is a round cylinder of soft iron in which are cut two deep grooves. It is wound with as many turns of wire as will fill the groove.

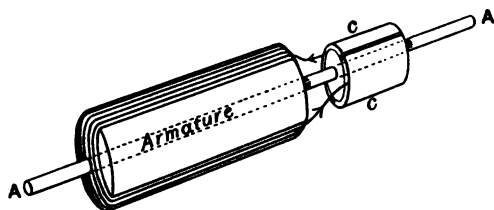


Fig. 82.

The Commutator. The ends of the wire are joined each to one half of a brass cylinder *cc*. These half cylinders are mounted on an insulating cylinder of ebonite through which the axle *AA* of the armature passes (fig. 82).

The Brushes, bb fig. 83. Two brushes of copper gauze are arranged to bear lightly on opposite sides of the commutator. The current enters by one and leaves by the other. If the brushes and commutator are arranged as

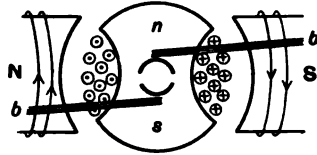


Fig. 83.

shewn in the figure it is evident that the direction of the current in the armature is reversed at each half revolution: the magnetism in the armature is reversed with the current. In the position shewn the lower half of the armature is a South pole: the upper a North. The armature will therefore be pulled round in the direction of the arrow till the line *ns* is horizontal.

The armature current now reverses, so that the left-hand side becomes North: it is pushed away from the North pole of the field magnet and attracted by the South so that the rotation is continued.

In these figures the motor is shunt wound, i.e. the windings of the armature and field magnet are in parallel. The current from the leads divides into two parts: one enters the field magnet, the other passes through the brushes and the commutator round the armature. The armature winding is a shunt to that of the field magnet.

This "*H*" or shuttle armature is simple and easily understood. It has however very bad faults. The sudden reversal of the armature current gives rise to sparking: the torque produced is very uneven: the machine will not start by itself in all positions.

CHAPTER XII

INDUCED CURRENTS

114. Currents induced by Magnets. We have seen several examples of magnetic effects produced by currents. Faraday in England and Henry in America worked at the converse problem of producing currents by means of magnets.



Fig. 84.

Take a coil of several turns of wire and join the ends of it by long leads to a delicate galvanometer. Bring the North pole of a magnet quickly up to the face of the coil. What effect has this on the galvanometer? When the needle has come to rest withdraw the magnet quickly and again notice the galvanometer. Repeat the experiment by bringing up the South pole. Note carefully the direction of the deflection in each case. In these experiments the galvanometer must be some little distance from the coil: otherwise the action of the magnet on the galvanometer needle will be sufficient to produce deflection.

115. Currents induced by currents. Two coils of wire are to be placed face to face. *A*, the *primary* as we shall call it, is to have in series with it a cell and key *K* by which circuit may be made or broken. *B*, the *secondary*, is to be connected by long leads to a galvanometer, *G*. (1) Start

a current in *A* by depressing the key: notice the galvanometer as you do so. (2) Break the current by releasing the key.

Place a bar of soft iron through the two rings, and repeat the experiment by making and breaking the circuit. What differences are observed?

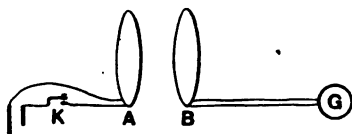


Fig. 85.

Find out the direction of the current in the primary *A*:

find out also that in the secondary *B* when *A* is (1) made, (2) broken.

Vary the experiment by keeping a steady current in *A* and suddenly separating the coils or bringing them nearer together. Since a current in a ring behaves exactly as a magnet, this variation of the experiment is only a repetition of that in Art. 114.

116. Faraday's ring. The apparatus with which Faraday in 1831 discovered the existence of induced currents was a ring of soft iron, about six inches in diameter. One side of this was wrapped with a coil of insulated wire the ends of which could be connected to a battery. The other side was wrapped with a second wire kept quite separate from the first. The ends of this were joined to a galvanometer. On connecting up the battery through the primary, Faraday found that a transient current was produced through the secondary: on breaking the primary there was another transient current in the opposite direction.

117. Lenz's Law. Suppose *AA* is a ring of wire in the plane of the paper. If the North pole of a magnet comes up to this from beneath along the axis of the ring, a current is induced. The preceding experiments shew that this current is in the direction indicated by the arrow. The ring with its current is equivalent to a magnet with its North pole downwards, its South upwards. It will therefore exert a repulsion on the approaching magnet and tend to check its motion. If however the magnet is withdrawn, the induced current is reversed, so that

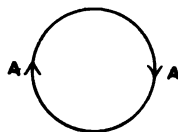


Fig. 86.

there will be a pull on the retreating magnet; again tending to check the motion.

These cases illustrate a general law enunciated by Lenz. "*The direction of an induced current is always such as to tend to stop the cause producing it.*"

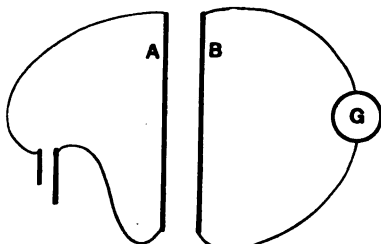


Fig. 87.

Again consider two wires *A*, *B* side by side. If *A* carries a current and is brought up nearer to *B*, there is, as we have seen above, a current in *B* in the opposite direction: so that (Art. 109) we have a repulsion between the conductors *A* and *B* tending to stop the approach. If on the other hand *A* is removed, the current in *B* is reversed (i.e. is now in the same direction as that of *A*), so that we get an attraction tending to check the separation.

The starting of a current in *A* has the same effect on *B* as the sudden approach of *A* to *B*; hence the starting of a current induces a reverse current in *B*; the cessation of the current in *A* induces a direct current in *B*.

118. The Induction Coil. The main parts of an induction (Rumkorff) coil are

(1) the core *AB*: a rod of soft iron or bundle of soft iron wires;

(2) the primary winding: this is a helix of thick wire wrapped round the iron core. The length of this is short and there are only a few layers;

(3) the secondary winding: this consists of a very large number of turns of fine wire: it may be half a mile or more in length. The ends *w*, *w* of this are joined directly to the terminals at the top of the coil. The wire must be carefully insulated. It is usually wound in several sections and mounted on an ebonite spool;

(4) a contact breaker. In the figure we shew a hammer break. A soft iron head H is attached to the spring S . The action is exactly similar to that of an electric bell. P, p are platinum studs. The hammer break is generally replaced in large coils by a motor or electrolytic break ;

(5) a condenser CC , the opposite sides of which are joined to the screw W and hammer in the break. It consists of a large number of sheets of tinfoil, insulated by waxed paper.

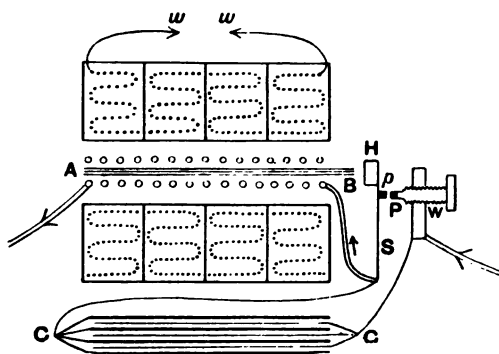


Fig. 88.

The action is as follows: When the current is switched on, it magnetises the core AB . This creates an induced current (or at any rate an induced E.M.F.) in the secondary. When the core is fully magnetised, the primary current is suddenly broken, for the head H is pulled over to the core and contact is broken at Pp . The sudden stoppage of the current in the primary gives rise to another induced current in the secondary: this will be in the direction opposite to that of the first induced current. Hence as the current in the primary is made and broken it induces a backward and forward E.M.F. in the secondary. The magnitude of the induced E.M.F. is dependent on the number of turns of wire in the secondary: the greater the number of turns the higher the E.M.F.

It is very important to remember that the currents induced in the secondary last only as long as the changes producing them last. When a steady state is reached in the primary, the current in the secondary ceases.

119. The action of the condenser. In an electric bell there is a little sparking at the platins: this is caused by the sudden breaking of the current. The current in an induction coil is very much larger than that required for a bell so that, unless some steps were taken to prevent it, the sparking at the terminals would be excessive and quickly wear the platinum away. When however the condenser is inserted it is no longer necessary for the current to "arc" across the gap: it may go to charge up the condenser instead.

The condenser has another effect: it enables the current in the primary to vanish quickly: it also causes the rise of the current when contact is renewed to be less rapid. In other words it accelerates the "break" and retards the "make." Now the E.M.F. in the secondary at any time varies with the *rate* at which the primary current is changing: it is therefore much greater at the break than at the make. If the E.M.F. produced at the make is insufficient to allow the discharge to pass a spark between the terminals of the secondary, we can only get a current at the break: this will always be in the same direction: it will obviously not be continuous but will pass in a series of pulses.

120. The Earth Inductor. This consists of a ring of large diameter wrapped with several turns of wire. It is so arranged that it can turn about a vertical diameter in its plane. The ends of the winding are connected to the terminals of a ballistic galvanometer.

Let one side of the coil face due South: then give a sudden half turn so that it faces North. How is the galvanometer affected? What effects do we get when the axis of the coil is turned (1) through four right angles, (2) from S. to W., (3) from S. to E., (4) from E. to W.?

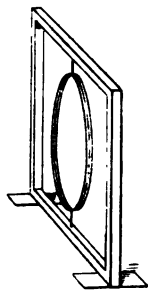


Fig. 89.

121. Faraday's Disc. A disc of copper, *C*, say five or six inches in diameter is turned rapidly about an axle *A* at right angles to its plane. A spring, *B*, which acts as a "brush" rests lightly against the edge of the disc. The disc is placed between the poles *N*, *S* of an electromagnet. Faraday found that a current flowed between the axle and the spring.

Notice that it is easy to turn the disc when the electro-magnet is not excited: and hard when it is excited. (Cf. Lenz's Law.)

If a sixpence is held up by a twisted thread, it will generally untwist itself and spin quickly, but if held between the poles of a powerful magnet, it will only untwist itself very slowly ; its rotation is "damped" and it seems to act almost as if it were placed in a viscous liquid. The reason of course is that currents are induced in the coin by its motion near the magnet: these currents give rise to forces tending to stop the rotation which causes them.

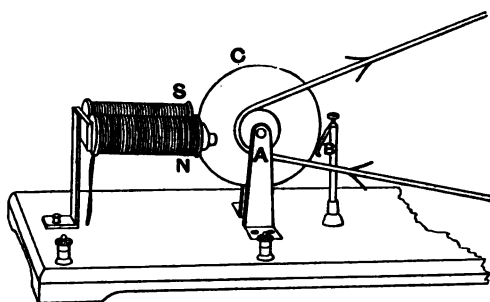


Fig. 90.

The suspended needle in a galvanometer comes to rest more quickly if a lump of copper is near to it than if no metals are present.

CHAPTER XIII

MAGNETIC FLUX

122. Lines of force. In Chapter I we have already said a few words about lines of force and described methods of tracing them. We are now going to carry the subject a little farther. Look first at fig. 129, end of book. In it we can see lines running between the N. and S. poles of two magnets. We should get a similar figure if the poles were joined by elastic strings which repelled one another. It is useful to imagine that such elastic strings exist and that they form the mechanism by which poles attract or repel one another. Whenever two poles attract, the lines of force always run from one to the other; when two poles repel, the lines of force do not unite them but run up against one another and seem to push apart the poles to which we imagine them attached. See fig. 128.

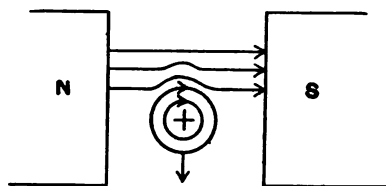


Fig. 91.

We have already given in Chapter I the lines of force due to a current in a straight conductor: also those due to vertical currents in the earth's field. Fig. 91 shews a similar case: that of a conductor carrying a current between the poles of a magnet. Notice that if the lines of force were elastic strings the conductor would be forced down in the direction of the arrow.

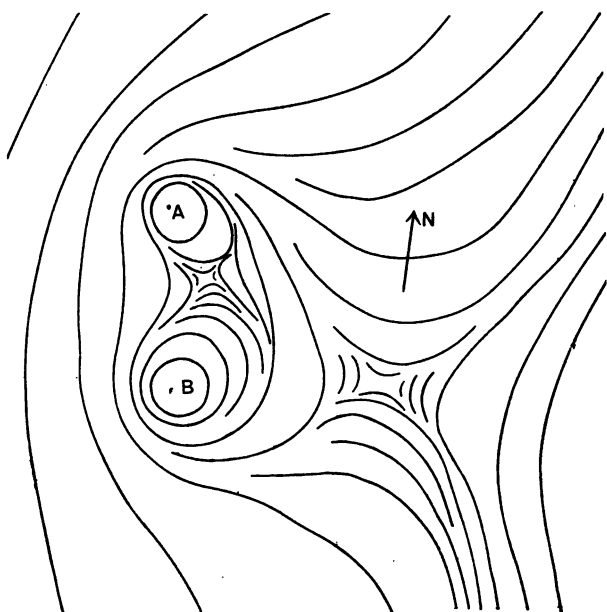


Fig. 92.

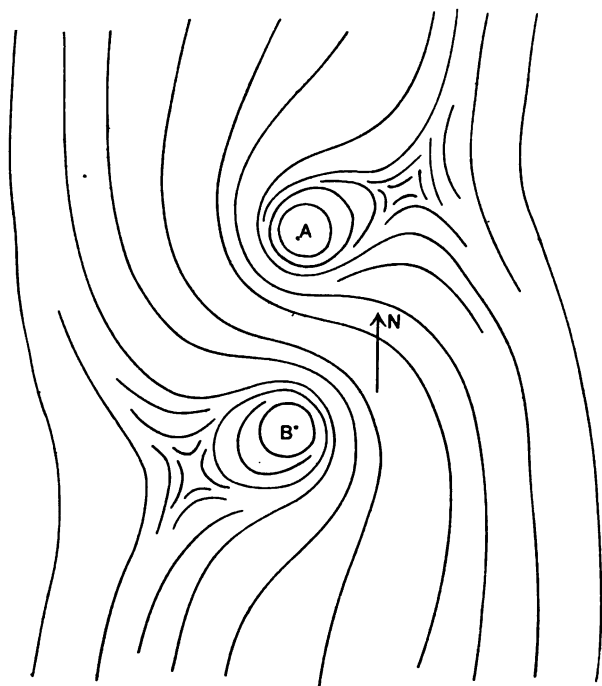


Fig. 93.

Fig. 92 represents the lines of force in a horizontal plane due to two vertical currents in the same direction; the earth's horizontal field affects these considerably; the lines indicate attraction.

In fig. 93, the currents are parallel but oppositely directed: in this case we have repulsion. Compare this figure with the lines of force due to a short bar magnet.

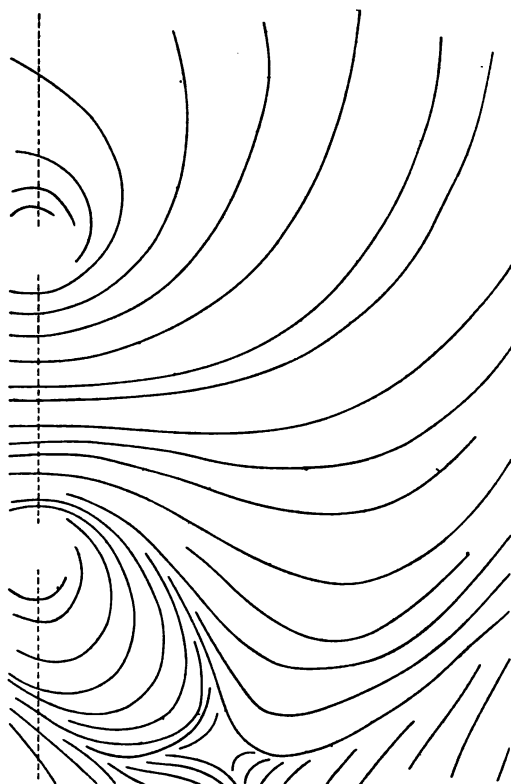


Fig. 94.

Fig. 94 shews the lines of force in half the field due to the current in a galvanometer ring placed in the meridian. With this compare fig. 127, Art. 147.

123. The number of lines of force: flux. At any point in a magnetic field we can draw a line in the direction

of the magnetic force there: hence through any area of finite size we could draw an infinite number of lines of force. It is convenient however to give a special meaning to the expression "the number of lines of force." We imagine (1) that every pole of unit strength has 4π lines attached to it, (2) that if an area $\mathbf{A}$ sq. cm. is at right angles to a uniform field of $\mathbf{H}$ gauss the number of lines passing through it is $\mathbf{AH}$. These two methods of defining the number agree—for suppose we had a pole of strength m : let us surround it by a sphere of radius r . On the first supposition the number of lines which emerge from the pole is $4\pi m$, these all cut the spherical surface. On the second supposition the strength of field at any point on the sphere is, by the law of the inverse square, m/r^2 , the area of the surface is $4\pi r^2$. Hence the total number of lines of force which cross the surface

$$= 4\pi r^2 \times m/r^2 = 4\pi m,$$

the same result as obtained by the other definition.

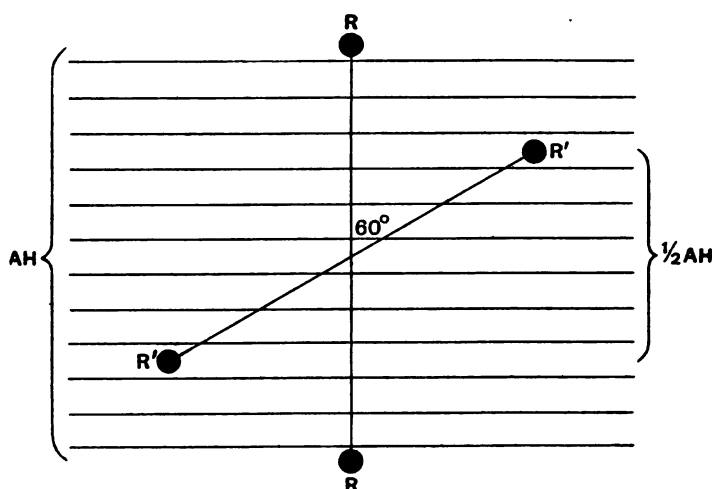


Fig. 95.

If we select an area which is not at right angles to the field, we define the number of lines as the product of the area and the component of the field perpendicular to the area. Thus suppose we have a ring RR of area $\mathbf{A}$ placed at right angles to a uniform field of strength $\mathbf{H}$, the number of lines

which thread the ring is $\mathbf{HA}$. But if the ring is turned through 60° into the position $R'R'$, the component of the field perpendicular to the coil is $\mathbf{H} \cos 60^\circ$, i.e. $\frac{1}{2}\mathbf{H}$. So that the number of lines threading the ring is only $\frac{1}{2}\mathbf{AH}$. This is shewn by the diagram. The number of lines is also spoken of as the *flux*, and the unit in which we measure it is called the *maxwell*. Thus the flux through a horizontal coil of 5 turns each of area 60 sq. cm., in a field the vertical component of which is .4 gauss, is $5 \times 60 \times .4$, i.e. 120 maxwells.

124. Induced E.M.F. Now whenever a ring of wire moves in a magnetic field in such a way that the number of lines of force passing through it is changed a current is always induced in it: this current lasts as long as the number of lines is changing: further it can be shewn that the E.M.F. which drives the current is proportional to the *rate* of change of the number of lines of force.

The measure in volts of the E.M.F. produced by the motion of a ring is the rate of change in the number of lines of force multiplied by 10^{-8} . As an example let us suppose that a framework of wire $ABCD$ has a metal bar FG resting across it. Let this be placed with BC vertical and BA pointing to the East. Then taking the earth's horizontal field to be .18 gauss, the number of lines which thread the area $BCGF$

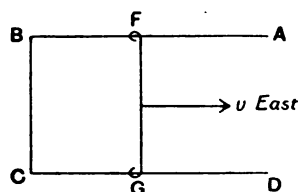


Fig. 96.

$$= .18 \times \text{area } BCGF = .18 \times BC \times CG.$$

If the bar slides along the wire with a velocity v cm. per second, then the size of the area is enlarged by vl sq. cm. in one second, l being the length in centimetres of BC .

$\therefore$ rate of change in the number of lines of force

$$= v \times l \times .18.$$

$\therefore$ the induced E.M.F. $= l \times v \times .18 \times 10^{-8}$ volts.

If the resistance of the circuit $BFGC$ is $\mathbf{R}$ ohms, the current induced will be $\frac{lv}{\mathbf{R}} \times .18 \times 10^{-8}$ amperes.

The electromagnetic unit of E.M.F. may be defined as the E.M.F. produced in a wire one cm. long by moving at one cm./sec. directly across a magnetic field of unit strength.

125. Direction of induced current. In which way does the current flow? There are several rules by which we can find this out.

(1) In fig. 96 the lines of force due to the earth run through the paper from face to back: if the number passing through the circuit is increasing, the lines of force due to the current must be passing in the other direction so that the current must be in the direction *GFBC*, for the tendency is always for the number of lines of force passing through an area to remain unaltered. If the slider had been moving to the West, the number of lines due to the earth through the circuit would have been diminishing: to keep this constant the current's lines of force would have to be added on in the same direction to tend to check the decrease. The current then in this case would be in the direction *GCBF*.

(2) A simpler way perhaps is this. Look down on the framework from above.

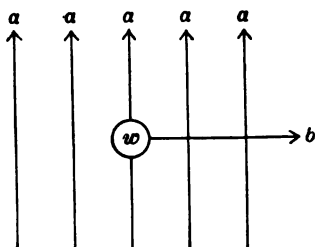
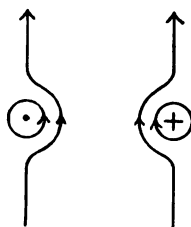


Fig. 97.



(i) Fig. 98. (ii)

In the diagram, fig. 97, *w* represents a section of the sliding bar *FG*; *b* the direction in which it is pulled, *a, a, a, ...* are the lines of force. Now Lenz's law tells us that the force on the conductor must tend to stop the motion, so that if the lines of force can be thought of as elastic strings they must pull to the West. The combination of the earth's field and the current field must therefore be like fig. 98 (i) and not like fig. 98 (ii).

Fig. 98 (i) is the field due to an approaching current; hence the current in the wire *w* is in the direction *GF*.

(3) Fleming's Dynamo rule. Point the **M**iddle **F**inger of the *left* hand parallel to the lines of **M**agnetic **F**orce, the **t**humb in the direction in which the wire is **t**hrust, then the

index (first) finger can point in the direction of the induced current.

126. Incomplete Circuits. We have seen that a current is generated in a closed circuit whenever the number of lines of force passing through it is being altered. The reason for this is that the circuit or at any rate a portion of it is cutting lines of force. If any conductor of any shape moves in such a way that it cuts lines of force, an E.M.F. is induced in it. This E.M.F. will not necessarily give rise to a current. Take the case of a key ring, facing North and falling in a vertical plane cutting the earth's lines of force. A little consideration shows that both the top half and the bottom half have in them an induced E.M.F. in the same direction which tends to drive currents from the West side to the East. These two halves are in parallel and may be compared to two cells joined in parallel. If we completed a circuit by joining the parts *A*, *B* by light flexible wires *w*, *w*, then we should get a current.

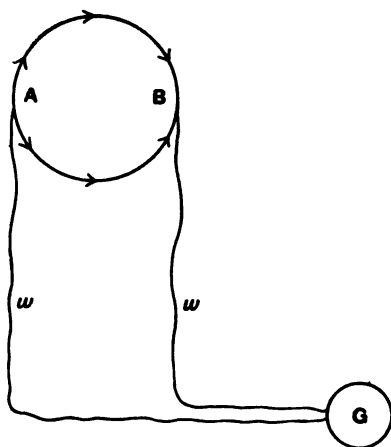


Fig. 90.

127. Faraday's Disc. One case of interest is Faraday's Disc, fig. 90. Suppose a copper disc is mounted on an axle pointing North and South so that the plane of the disc is at right angles to the meridian. Suppose we are looking at the wheel from the South, it is seen to be rotating clockwise. There will then be an electromotive force along all the radii tending to drive a current from the circumference to the centre. The magnitude of this E.M.F. can be easily calculated. Suppose *H* is the strength of the field normal to the disc, *r* the radius, *n* the number of revolutions per second. Then every second

each radius sweeps out an area of $\pi r^2 n$, and therefore cuts $\pi r^2 n H$ lines of force. The induced E.M.F. is therefore $\pi r^2 n H \times 10^{-8}$ volts. Of course no current flows unless the centre is joined to the circumference by a conductor which does not rotate with the disc.

EXAMPLES

1. Explain the fact that a wire spiral tends to contract when a current is passed through it. How can this tendency be increased? (Roget's Spiral.)

2. A gutta-percha covered copper wire is wound round a wooden cylinder. How would you wind it back (1) so as to increase, (2) so as to diminish, the magnetic effects which it produces when a current is passed through it? Illustrate your answer by a diagram.

3. A current is flowing through a rigid copper rod. How would you place a small piece of iron wire with respect to it so that the iron may be magnetised in the direction of its length? Assuming the direction of the current, state which end of the iron will be a north pole.

4. What mechanical and electrical effects would there be between two parallel circuits (*v.* fig. 77) in one of which a current is being constantly made and broken?

5. Explain exactly the effects of introducing a metal tube between the primary and secondary windings of an induction coil.

6. In a primary circuit flows a current which alternately increases and decreases. Near it is a secondary: will there be any attraction or repulsion between the two?

7. Two discs separated from each other are mounted on the same axis on which both are free to turn. If one is copper and the other is magnetised, what effect will be produced in the former when the latter is spun rapidly? (Arago's Disc.)

8. Why will a penny not spin when placed in a strong magnetic field?

9. The north-seeking pole of a very long magnet is brought from a great distance up to the centre of a small copper ring. What mechanical and electrical effects are produced? Upon what circumstances does the magnitude of each depend, and what is the connection between them?

10. An iron hoop is held in the magnetic meridian and is allowed to fall over towards the east. Explain why an electric current traverses the hoop, and state whether the current would flow north or south in the part of the hoop which touches the ground.

11. It is found that when an induction coil is being worked by a battery the current through the primary is much stronger if the circuit of the secondary is kept open than if it is closed. Account for this.

12. The time during which a magnet oscillates is decreased by placing it near a mass of metal. Explain this.

13. In what ways can a hoop of wire be moved if no currents are to be induced in it?

14. If a watch were composed wholly of non-magnetic material, would it be influenced by the presence of a magnet? Give reasons.

15. A magnet is brought up and passed through a coil of wire. What currents will be induced?

16. Would the oscillations of a galvanometer with an astatic combination be damped by the presence of a lump of copper?

17. A helix of wire surrounds a vertical iron pillar on the top of which a disc of copper is placed. When an alternating current is passed through the wire, what will be the effect on the disc?

18. The axle of a railway carriage (length l) is moving with a velocity V in a place where the vertical component of the earth's field is F ; find the electromotive force produced.

If $l = 1.5$ m., $V = 1666$ cm. per sec. and $F = .44$, find the E. M. F. in volts.

19. Shew how to determine approximately the direction of the magnetic meridian at a place by experimenting with a coil of wire moveable about a vertical axis, and connected with a sensitive galvanometer.

20. A horizontal copper disc 20 cm. in diameter is set spinning about a vertical axis through its centre with a speed of 100 revolutions per second in the earth's magnetic field. What is the nature of the distribution of potential in the disc? Find the value of the difference of potential in volts between the edge and the centre of the disc, if the vertical intensity is 0.50 gauss.

21. What is the flux through a ring of radius 5 cm. placed 25 cm. from the centre of a magnet of moment 300? The magnet points to the centre of the coil and is at right angles to its plane.

22. The rails of a tramway are a metre apart, insulated from the ground but joined at the terminus. A tram is travelling towards the terminus at 20 km. per hour. If the resistance of the circuit of rails and axle is 10 ohms, find (1) the current produced; (2) the work done per second in producing this current; and (3) the extra force required to draw the tram. Assume $H = 0.2$, dip $= 60^\circ$.

23. The resistance of the circuit of a Faraday's disc is 0.25 ohm, the radius of the disc 8 cm., the strength of the field 3000 gauss. Find the current produced when the disc revolves 1500 times per min.

24. Draw the lines of force for a galvanometer ring with its plane in the meridian.

25. A rubber tube of section not quite uniform is filled with mercury. Would the passage of a strong current through it produce any mechanical effects?

26. If an alternating current were passed through the primary of an induction coil, how would the current produced in the secondary differ from the current induced by a direct current with hammer break?

27. Make a drawing of a break for an induction coil in which the current is broken by a motor-driven dipper which oscillates in and out of a vessel of mercury, the surface of which is covered with alcohol. Shew the connections to the primary and condenser of the coil.

28. Two plain coils of wire wound in the same direction are joined in series; they can rotate freely about vertical diameters. Explain what effects the rotation of one will produce on the other.

29. If a strong current is driven by means of a cell through the two coils (in Ex. 28), what positions will the coils take up?

30. Water in a hollow anchor ring surrounding an electromagnet can be made to boil by passing a strong alternating current through the winding: Explain this effect. Of what material would you make the anchor ring?

CHAPTER XIV

DYNAMOS AND MOTORS

128. Simple Alternating Dynamo. We have seen that if a coil of wire is rotated in a magnetic field we get currents induced in it passing first in one direction, then in the other. Consider the arrangement in fig. 100. $ABCD$ is a

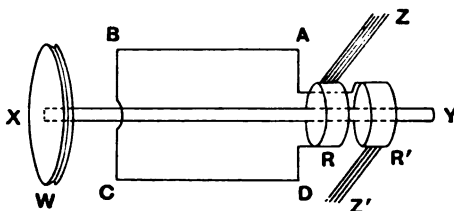


Fig. 100.

coil of wire nearly in the plane of the paper: we have only drawn a single turn: in practice we should have a great many. The ends of the wire are joined to two rings of brass or copper RR' , mounted on, but insulated from, a steel axle XY . The whole is rotated by a belt passing over the driving pulley W . Resting lightly on the rings R, R' are two copper-gauze or carbon brushes Z, Z' . They are joined to the circuit in which current is required.

The coil is mounted between the poles N, S of a magnet: this is shewn in fig. 101. As the coil revolves we have alternating currents induced in it; for during half a rotation the current passes from Z and R round $DCBA$ to R' and Z' and during the other half it will go through the same circuit in

exactly the opposite direction. A current such as this has no effect on a compass needle: it is however just as useful for lighting lamps and heating as a direct current. In practice the coil $ABCD$ is wound on a soft iron cylinder or shuttle mounted on the axle.

A "magneto" similar to this is used on the telephone "call."

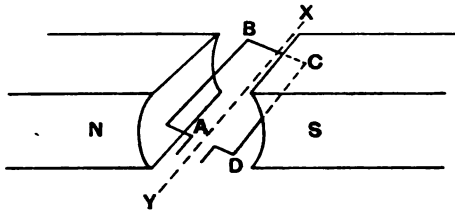


Fig. 101.

129. Direct Current Dynamo. The alternating dynamo may be turned into a direct current instrument if we remove the rings and replace them by a commutator, fig. 102. The machine would then be exactly like the motor in Art. 113. Observe that when the field magnet has the

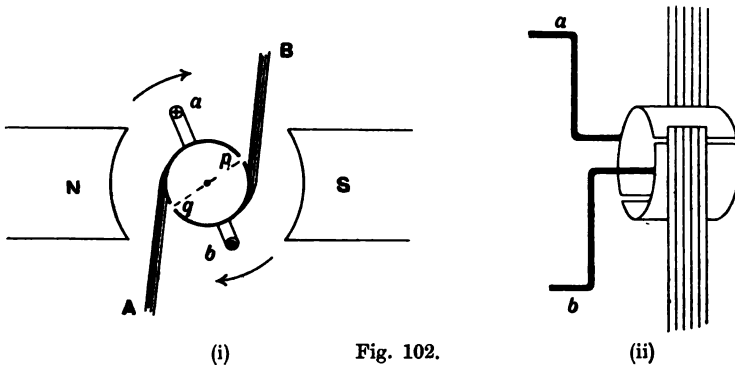


Fig. 102.

polarity indicated and the armature is rotated in the direction of the arrow, there will always be an E.M.F. down the conductor (a) which happens to be on the left-hand side and up that on the right (b). The current must therefore come from the brush B round the outside circuit, back to A , down a and up b . The brushes must be so arranged that the dividing line pq is under the brushes at the moment that the winding

α, b is at right angles to the field N, S . There is no need for NS to be a permanent magnet: it may be made of soft iron or steel and excited by the current generated in the armature.

The current given by this shuttle-wound dynamo is always direct but fluctuates. It has its maximum value when the plane of the coil is parallel to the field, its zero when it is at right angles to it. Fig. 103 represents the relation between the current and the angular position of the coil.

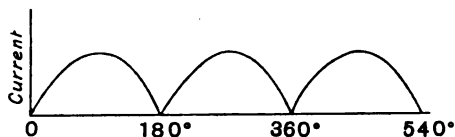


Fig. 103.

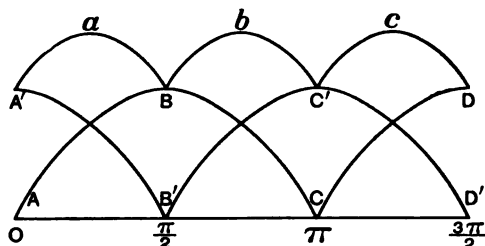


Fig. 104.

Now suppose it were possible to have two windings on the same armature and that these were so arranged that while one gave the maximum current the other was at the zero.

The currents given by the first and second windings would be represented by the curves $ABCD, A'B'C'D'$.

The current given by the two together would be represented by a curve the ordinate at which at any point was equal to the sum of the ordinates of $ABC... A'B'C'$ at the same point. The curve $A'aBbC'cD$ will therefore represent the total current.

The current still fluctuates but very much less than before. In an ordinary dynamo we have a great number of different windings and so obtain a current which is almost without fluctuations.

130. Ring Armature. Fig. 105 is supposed to represent a section of a ring-wound cugged armature: eight windings are shewn. If the armature were at rest the lines of force would not pass straight from the North pole of the field magnet to the South, for they always run in soft iron rather than in air: their course is shewn by the filings in fig. 133.

From this it will be seen that all conductors on the outside of the field cut lines of force as they pass round, but the conductors on the inside do not, so that only half the wire in armature is useful for generating electromotive force.

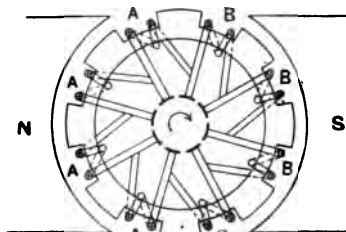


Fig. 105.

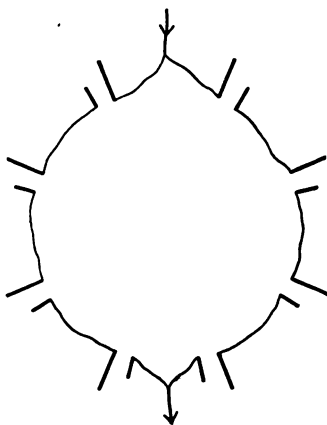


Fig. 106.

All the outside conductors on the left-hand portion of the armature have a downward E.M.F. (i.e. downward as seen in the figure), those on the right-hand side are upward. Each of the four coils *AAAA* therefore tends to drive a current along the winding in the direction shewn by the crosses of the arrows. They are therefore equivalent to four cells joined in series. The coils *BBBB* on the other side form another set of four in series: they are joined to the other four in parallel. The machine may therefore be compared with a battery of eight cells joined in two parallel rows, each row containing four cells in series. Cf. fig. 106.

131. Drum winding. In a simple drum-wound armature the wires pass down one side of the soft iron core and up nearly opposite. The diagram (fig. 107) gives a winding of an armature with eight slots. The arrow points and tails indicate the direction of the induced E.M.F. The continuous lines shew the winding at the commutator end of the drum; the dotted lines mark the connections at the other end. Notice that here again we have two courses through the armature winding from brush to brush. The two courses are in parallel and each consists of four windings in series.

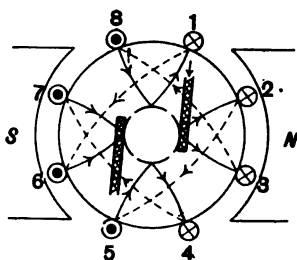


Fig. 107.

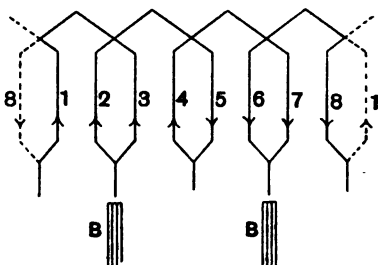


Fig. 108.

In fig. 108 is shewn a developed diagram of the same winding. It is the figure we should obtain if we imagined the cylinder on which the wires are wound to be slit parallel to the axle and spread out flat.

132. Field magnet windings. In a dynamo the field magnet may be (1) a permanent magnet, (2) separately excited, (3) series wound, (4) shunt wound, (5) compound wound.

(1) A permanent magnet is seldom found except on toy machines and in the call of a telephone.

(2) The field magnet of a dynamo used to give alternating currents is excited by a current taken from a direct current dynamo.

(3) In a series dynamo, fig. 109 (a), the current from one brush passes round the field magnet and outside circuit in series back to the other brush. All the current passes round the field magnet, armature, and circuit. It is often hard to get such a dynamo to start.

(4) In a shunt wound machine, fig. 109 *b*, the winding of the field magnet is in parallel with the outside circuit. The current generated in the armature divides into two parts: one goes round the field magnet, the other round the external circuit.

(5) In a compound wound dynamo, fig. 109 *c*, there are two windings on the field magnet. One of these is a shunt to, the other in series with, the external circuit.

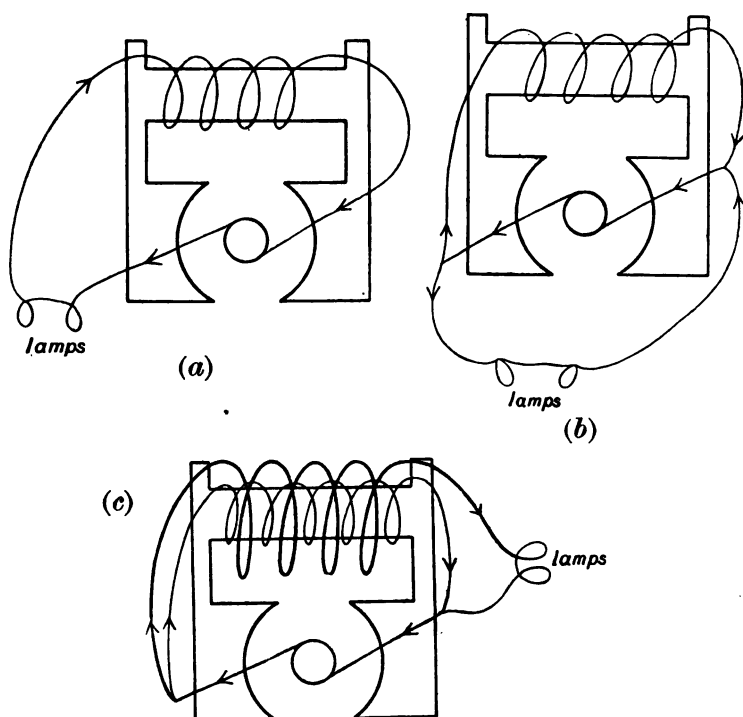


Fig. 109.

If the armature of a dynamo is rotated at a constant speed the terminal voltage generally increases with the current taken in the external circuit if the field magnet is series wound: it decreases for a shunt wound. In a compound wound machine, the volts should be almost independent of the current used.

For charging secondary cells we cannot use a series wound dynamo. If we tried to do so, there would be a danger of the current reversing in the field magnet and so altering its polarity. If this occurred the dynamo would help the cells to discharge. This would not be the case in a shunt wound dynamo.

133. Motors. Any dynamo described can be used as a motor, but since the current in the armature is only kept low by the back E.M.F. generated by its rotation in a magnetic field, the current must not be turned full on till the motor is going full speed. Also remember that a shunt wound motor must not be loaded till it is going full speed. A series wound motor will start from rest under load.

134. Heating. When a dynamo has been running for some time its temperature rises. There are several reasons for this.

- (1) Mechanical friction at the bearings.
- (2) The currents in the windings of the armature and field magnet warm these conductors.
- (3) Eddy currents. Not only do useful currents flow through the copper windings but currents are also induced in all the moving parts of the machine. In the body of the armature these are reduced by building it up in soft iron discs, insulated from one another and mounted on the axle.

The motion of the magnetised armature also induces currents in the stationary field magnet: it is not usual to take any steps to prevent these.

- (4) Hysteresis. The armature is magnetised in different directions as it passes between the poles of the field magnet. Now to magnetise a piece of iron, to demagnetise and then reverse the magnetisation requires a certain amount of work. The work done in taking a piece of iron through a magnetic cycle appears in the form of heat. It is impossible to avoid this source of heat but the effect is much less in some kinds of iron than in others, *v. Art. 175.*

135. Rotating Coil. To find the E.M.F. induced in a coil rotating in a uniform field.

Let A = area of coil.

Let n = number of turns.

$\mathbf{H}$ = strength of the field.

ω = angular velocity of the coil about a diameter in its own plane perpendicular to the lines of force.

θ = the angle the normal to the plane of the coil makes with the lines of force at any time.

Then the total number of lines of force which thread the windings of the coil is $\mathbf{A}n\mathbf{H} \cos \theta$.

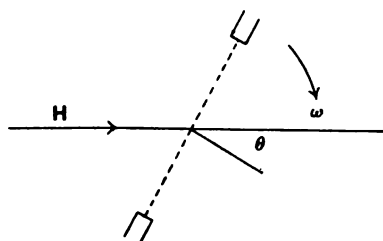


Fig. 110.

In a short time, dt , the coil turns through an angle $d\theta$ which is equal to ωdt . The change in the number of lines of force is

$$\begin{aligned} \mathbf{A}n\mathbf{H} \{ \cos \theta - \cos (\theta + d\theta) \} \\ = \mathbf{A}n\mathbf{H} \sin \theta d\theta \\ = \mathbf{A}n\mathbf{H} \sin \theta \cdot \omega dt. \end{aligned}$$

Hence the rate of change is $\mathbf{A}Hn\omega \sin \theta$: this expression measures the E.M.F. in C.G.S. electromagnetic units. If the resistance of the circuit is $\mathbf{R}$ (ohms) then the current in amperes will be

$$10^{-8} \frac{\mathbf{A}H}{\mathbf{R}} n\omega \sin \theta.$$

Note. (1) Since $\sin \theta = 0$ when $\theta = 0$, there is no E.M.F. when the coil is at right angles to the field: i.e. when the number of lines of force is a maximum, for then the *rate of change* in the number is zero.

(2) The expression for the E.M.F. is greatest when $\theta = \frac{\pi}{2}$, so that the E.M.F. is greatest when no lines pass through the coil, for then the rate of change is greatest.

(3) The E.M.F. changes sign when $\theta = 0, \pi, 2\pi$, etc., i.e. the E.M.F. reverses whenever the coil is at right angles to the field.

The charge which passes in any time is of course equal to the product of the average value of the current and the time,

i.e. charge = $\frac{\text{average rate of change in number of lines}}{\text{resistance}} \times \text{time}$,

but the average rate of change of any quantity $\times$ time = total change in that quantity. Hence the charge which passes in any time

$$= \frac{\text{total change in number of lines of force}}{\text{resistance}}.$$

If we measure the resistance in ohms, we must multiply this expression by 10^{-8} , if we wish to express the result in coulombs.

136. Earth Inductor. Let us apply this result to the case of the earth inductor (Art. 120). Suppose the inductor consists of n turns of wire of radius a cm., and that the total resistance of inductor and galvanometer is R ohms. When the coil is at right angles to the earth's field the horizontal component of which is H (18 dyne) the number of lines of force passing through it will be

$$Hn \cdot \pi a^2.$$

If then the coil is turned through 90° so that it now faces East and West, no lines will pass through it. Hence the charge driven round the circuit = $\frac{\pi a^2 n H}{10^8 R}$.

If it had been turned through 180° the charge would have been double this: but if through 360° , it would have been zero, for the current would have reversed during the second half of the revolution.

137. Self-induction. When the current round a circuit is altering we have seen that an induced E.M.F. is generated which tends to check that change and that the magnitude of this is proportional to the rate of change.

Now consider a circuit through which unit current threads L lines of force: suppose that originally there is no current

and that an E.M.F. of $\mathbf{E}$ is suddenly applied: denote the current at any time t by i .

The rate of change of current will be di/dt and the back E.M.F. induced is $\mathbf{L}di/dt$. The real E.M.F. will therefore become $\mathbf{E} - \mathbf{L}di/dt$.

If $\mathbf{R}$ is the total resistance, then by Ohm's Law

$$\mathbf{E} - \mathbf{L} \frac{di}{dt} = i\mathbf{R},$$

and the solution of this equation is

$$\mathbf{E} - i\mathbf{R} = e^{-\frac{\mathbf{R}}{\mathbf{L}}t},$$

$$\text{i.e. } i = (\mathbf{E} - e^{-\frac{\mathbf{R}}{\mathbf{L}}t})/\mathbf{R}.$$

Hence we see that the current does not rise immediately to its final value $\mathbf{E}/\mathbf{R}$.

The quantity $\mathbf{L}$ is termed the 'inductance' or coefficient of self-induction of the circuit. The coefficient of mutual induction of the circuits is the number of lines of force passing through either, due to unit current in the other.

In all our previous results we have neglected inductance. If the coil in Art. 135 had an appreciable coefficient of self-induction the expressions for charge and current would have to be much altered.

EXAMPLES

1. A coil of 25 turns, radius 50 cm., lies flat on a table at a place where the vertical field is 4 dyne. It is connected to a ballistic galvanometer and the total resistance of the circuit is 3 ohms. What charge will pass through the galvanometer if the coil is (1) reversed, (2) placed in the meridian?

2. What is the flux through a coil of area 50 sq. cm. and 40 turns when lying on a table? Assume values given in Art. 6. If the terminals of the coil are connected to a galvanometer and the total resistance of the circuit is 5 ohms, what charge will pass through the galvanometer when the coil is reversed?

3. What is the flux through a ring of radius 15 cm. due to a magnet pole of strength 3000 placed on its axis and 25 cm. from the centre?

4. Would aluminium wire be suitable for winding motors?

5. What points would you have to consider before deciding what thickness of wire you would use to wind (1) the armature, (2) the field magnet of a dynamo?

6. If the direction of the current supplied to a motor is reversed, will the direction of revolution be reversed?

7. Draw the developed winding of a dynamo armature with sixteen slots. (Join 2 to 9 behind, 9 to 4 in front.) Draw also a diagram similar to fig. 108.

8. A hoop of wire is made to revolve about a fixed vertical diameter. Describe how the direction and magnitude of the currents induced change during one complete revolution.

9. What causes tend to make masses of iron used in dynamos grow hot when at work? How is each cause best guarded against?

10. A current from a battery is passed through an ammeter and motor. Describe and explain the difference in the readings of the ammeter when the motor is (1) prevented from rotating, (2) allowed to run without load.

11. The motors of a tramcar are so connected that they may be joined up in series or parallel. What advantage does this arrangement offer and when must the driver use them in parallel and when in series?

12. When is a starting resistance necessary in a motor?

13. A dynamo is turned by a steam engine. In what practicable ways would it be possible to vary the voltage between the terminals?

14. How would you expect the voltage of a dynamo to be affected by the speed of rotation when the field magnet is (1) separately excited, (2) shunt wound?

15. What forces act on the wires in the armature of a dynamo? Are there any forces on the field magnet winding?

16. A coil of 50 turns of wire, 20 cms. in diameter, is mounted on a vertical axis and rotated at a uniform rate of 5 revolutions a second in the earth's magnetic field. ($H=18$.) The average current through the coil, irrespective of sign, is found to be 1.5 milli-amperes. Calculate the resistance of the coil in ohms.

17. A current C flows in a circuit of resistance R and inductance L . If the E. M. F. which drives this current is suddenly removed, shew that after a time t the current i is given by the equation $i = Ce^{-\frac{R}{L}t}$.

18. Prove that a current C in a coil of self-induction L has energy $\frac{1}{2}LC^2$.

19. Shew by differentiation that if the coil in Art. 135 has a coefficient of self-induction L the current at the period t is given by the relation

$$i = H \cos(\omega t - a) / \{R^2 + L^2\omega^2\}^{\frac{1}{2}},$$

where

$$\tan a = L\omega/R.$$

CHAPTER XV

AMPÈRE'S LAW

138. The magnetic force at a point due to a current.

Suppose a current i flows along a small length $PQ (= ds)$ of a conductor, that O is a point distant r from P and that PQ makes an angle θ with OP . Then the force on unit pole at O is $ids \sin \theta / r^2$ and is perpendicular to the plane of POQ .

If i is measured in C.G.S. electromagnetic units, and r in centimetres, then this force is measured in dynes. The result is known as Ampère's Law.

(1) Force at the centre of a circular coil.

Here each element of the coil is at right angles to the radius: hence the force will be simply

$$\int \frac{ids}{r^2} = \frac{i \times \text{length of coil}}{r^2} = \frac{2\pi n}{r},$$

if r is the radius and there are n complete turns.

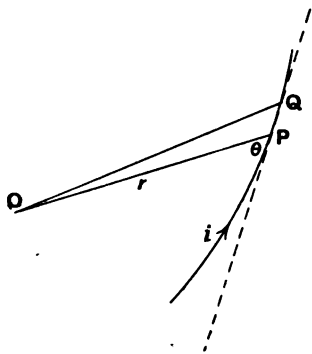


Fig. 111.

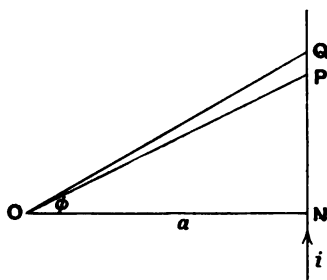


Fig. 112.

(2) Force due to a long straight current (fig. 112).

Suppose a point O is distant a from an infinite straight

conductor, N the foot of the perpendicular from O , PQ a small element of the conductor. Denote the angle NOP by ϕ , POQ by $d\phi$,

$$NP = a \tan \phi; PQ = d(a \tan \phi) = a \sec^2 \phi d\phi.$$

Magnetic force at O due to current i in element PQ

$$= \frac{iPQ \cos \phi}{OP^2} = \frac{ia \sec^2 \phi \cos \phi d\phi}{a^2 \sec^2 \phi} = \frac{i \cos \phi d\phi}{a},$$

$$\therefore \text{total force at } O = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{i \cos \phi}{a} d\phi = \frac{2i}{a}.$$

A deduction from this proposition is important. Suppose a magnet ns is fixed to an arm PQ which is capable of rotating about a vertical axis along which a current flows.

If r, r' are the distances of the poles of the magnet from the axis, then the moments about the axis of the forces exerted on the two poles are

$$\frac{2i}{r} \times r \times m \quad \text{and} \quad \frac{2i}{r'} \times r' \times -m,$$

if m is the strength of each pole.

Hence the total moment is zero, so that the magnet as a whole and therefore the arm as well, have no tendency to revolve round the conductor.

The result has been tested carefully by experiment and no trace of any tendency to rotate has been discovered. This is strong evidence of the truth of the law that the magnetic force due to a straight current varies inversely as the distance and also of Ampère's law from which it was deduced.

(3) Force at a point on the axis of a coil.

Suppose the coil has a radius a and that l is the distance of a point O on the axis AO from the coil CC' . Denote the angle COA by α . The force at O due to an element ds of the ring at C is ids/l^2 for the element is at right angles to OC . The direction of it is perpendicular to OC and its component

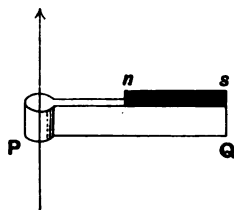


Fig. 113.

along the axis is $\frac{id\mathbf{s}}{r^3} \sin \alpha$, i.e. $\frac{ia ds}{r^3}$. But the resultant must by symmetry be along the axis: hence this resultant

$$= \int \frac{ia ds}{r^3} = \frac{ia}{r^3} \int ds = \frac{2\pi na^2}{r^3} i,$$

if the coil consists of n complete turns.

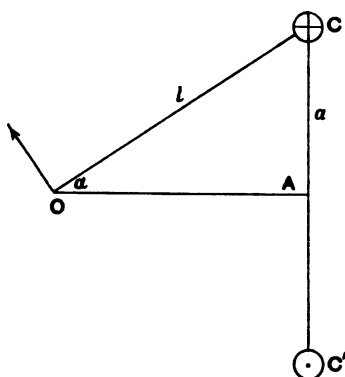


Fig. 114.

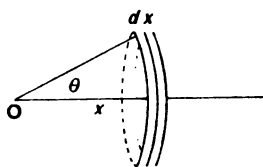


Fig. 115.

(4) Force on axis of a helix.

We shall only deal with the case in which the helix or solenoid is so long that we can suppose it infinite.

Assume that there are n turns per unit length, a is the radius. Consider the force at O due to the current i in a turn of wire distant x from O . Its magnitude will be

$$\frac{2\pi a^2}{(a^2 + x^2)^{\frac{3}{2}}} i.$$

In a length dx there will be $n dx$ turns of wire: hence the force due to the current in the length dx of the solenoid is

$$\frac{2\pi a^2}{(a^2 + x^2)^{\frac{3}{2}}} i n dx.$$

Hence total force

$$\begin{aligned} &= \int_{-\infty}^{\infty} 2\pi n a^2 i \frac{dx}{(a^2 + x^2)^{\frac{3}{2}}} \\ &= \int_0^{\pi} 2\pi n a^2 i \frac{a d\theta}{\sin^2 \theta} \frac{1}{a^3 \operatorname{cosec}^3 \theta}, \text{ if } x = a \cot \theta \\ &= - \int_0^{\pi} 2\pi n i \sin \theta d\theta \\ &= 4\pi n i. \end{aligned}$$

139. Work done in passing unit pole through a circuit. We shall only take a simple case : suppose the unit pole is passed through a circular current along the axis. The force on unit pole at a distance x from the centre of a coil (radius a) in which a current i is flowing is $\frac{2\pi a^2 i}{(x^2 + a^2)^{\frac{3}{2}}}$, and the work done in bringing the unit pole nearer by an amount dx is obtained by multiplying this expression by dx .

Hence the total work in threading the ring by passing unit pole from a great distance one side to a great distance the other side is

$$\int_{-\infty}^{+\infty} \frac{2\pi a^2 i dx}{(x^2 + a^2)^{\frac{3}{2}}} = 4\pi i.$$

This is only a particular example of a general theorem : the work done in carrying unit pole by any path round any conductor and returning to the starting point is $4\pi i$ where i is the current in the conductor.

140. Mechanical Force. A conductor carrying a current in a magnetic field is subject to mechanical forces. This fact has been illustrated in Chapter I, where a toy motor was described. The magnitude of the force acting on any element of the conductor is proportional to (i) the length of the element, ds ; (ii) the strength of the field, $\mathbf{H}$; (iii) the strength of the current, i ; (iv) the sine of the angle between the directions of the field and the element, ϕ . If all these quantities are measured on the electromagnetic system the measure of the force in dynes is $\mathbf{H}i \sin \phi ds$. The direction of the force is at right angles to both the field and the conductor. If the *middle finger* of the right hand is held parallel to the magnetic field, the *index finger* to the current (i), then the *thumb* when at right angles to both is pointing in the direction of the *thrust*.

EXAMPLES

1. Prove from Ampère's law that the work done in carrying a unit pole round a long straight wire carrying a current i is $4\pi i$.
2. Assuming the result of Art. 139, prove that the force at any point inside a long solenoid carrying a current i with n turns per cm. is $4\pi ni$ and that the force outside it is everywhere zero.

3. In fig. 10 the distance from the null point to the centre of the wire was 15 cm. and the strength of the earth's field was '185. Find in amperes the strength of the current.

4. A long solenoid of diameter 3 cm. is made of wire wrapped 8 turns to the cm. It carries a current of '53 ampere. What is the flux through it?

5. Shew that a circular coil of area a carrying a current i exerts on unit pole on its axis a force equal to that exerted by a magnet of unit length in "Gauss A" position (Art. 166) of intensity i and pole area a .

6. Two long straight parallel conductors, $2d$ apart, carry equal currents i : find the force at points equidistant x from both.

7. Find the total force exerted on a coil of radius r and n turns carrying a current i when a single pole of strength m is at the centre. Shew that this force is equal to the force exerted by the current on the pole.

8. Two parallel currents i, i' flow along conductors x cm. apart. Prove that the mechanical force per unit length which each exerts on the other is $2ii'/x$.

9. Two small coils on the same axis carry a current i . If the radius of each is r and the distance between them is d , find the force each exerts on the other.

10. Find the force on a long straight wire carrying a current i due to a magnetic pole of strength m at a distance d . Deduce this from Art. 140 by integration.

11. Find the resultant force on a magnet of moment M , length l , with the centre distant r from a conductor carrying a current i and placed as in fig. 113.

12. A coil of effective area a and radius r rotates with angular velocity ω about a vertical diameter. At the centre is mounted a compass needle which is deflected from the meridian through an angle δ by the current induced. Prove that the resistance of the coil in electromagnetic units is $\omega a^2 r^{-3} \cot \delta$. (B.A. method for determining the ohm.)

13. On the axis of a ring of 15 turns, radius 8 cm. and resistance '13 ohm is a magnetic pole of strength 3000. When the distance is x cm., what is the flux through the ring? Consider x great compared with radius. If the pole approaches with a velocity v cm./sec., find the current induced. Evaluate these when $v=20$, $x=200$.

CHAPTER XVI

ELEMENTARY MAGNETISM

141. Kelvin's Compass. The ordinary compass is a magnet supported in such a way that it can turn in a horizontal plane about a vertical axis. On the delicacy and workmanship of the support the value of a compass is dependent. On board ship a compass should be steady: that is to say its period of vibration should be long. It should be light, free from friction, and though the dial card should be large the needle should not be long. The Kelvin compass shewn in the figure satisfies all these conditions. It consists of eight

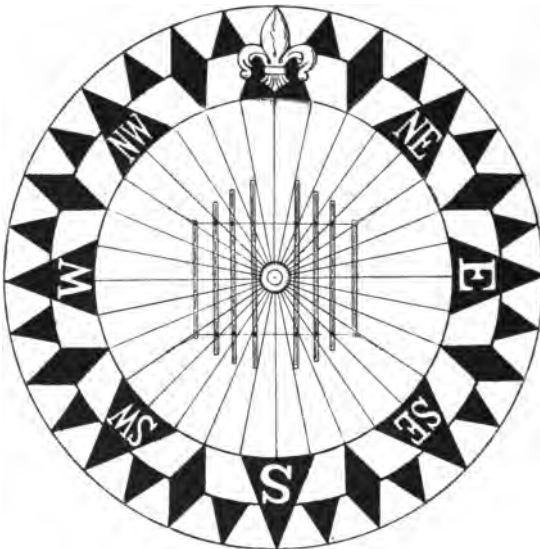


Fig. 116.

fine magnetised needles mounted parallel to one another, tied together with silk and mounted by silk stays in the middle of a light ring of aluminium on which are marked the cardinal points. The cup is of brass and holds a sapphire crown. The whole is supported on a hard smooth pointed pin (fig. 117).

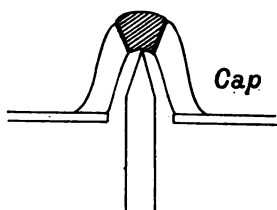


Fig. 117.

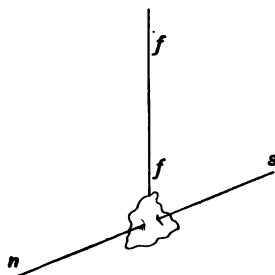


Fig. 118.

142. Action of Pole on Pole. Take a bar magnet: it will probably be marked on one end with an N. This should be its North Pole, but as the magnetism of such a bar is frequently reversed it is just as well to make certain of the point. Do this by holding it up by a piece of thread and a paper stirrup.

Now bring the end which you know to be North near the North Pole of a compass needle. In place of an ordinary compass, you may use a magnetised darning needle (*ns*) which pierces a piece of paper suspended by a length of silk fibre (*ff*), fig. 118.

The pole, *n*, moves away: indicating that the two North Poles repel one another. In exactly the same way you can shew that two South Poles can repel: that a North Pole attracts a South Pole.

It is near the *ends* of a magnet that the effects are most marked. To shew this roll a magnetised bar of steel in a box of iron filings. They will adhere in bunches round the ends: none will stay at the middle. We may regard the centres of these two bunches as the poles.

143. Magnetic Induction. A magnet attracts iron filings; a rough way of testing a piece of iron to see whether it is magnetised or not is to dip it into a box of iron filings. If the iron picks up the filings, it is magnetised. Test in this

way a few pen nibs or sewing needles and pick out a couple which shew no signs of magnetism. Hang both of them by their points on one end—say the North—of a strong magnet close together but not touching. Look carefully to see whether the two hang vertically side by side or the lower ends are farther apart than the points. In the latter case how could you possibly account for the apparent repulsion between the two?

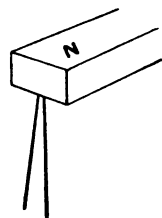


Fig. 119.

Test the two nibs again with the iron filings.

As the result of this experiment you will probably find that both nibs have become magnetised. This is not necessarily the case for all nibs are not exactly similar. If they are, test their polarity, i.e. see at which ends the North and South poles lie.

Take the magnet again and hang from the end a nib or a wire nail. At the end of this you will find you can hang a second, and so on until you get a string of perhaps four or five.

The formation of this chain of nails is easily explained. Suppose the North end of the magnet is used: then the first nail, when near it or in contact with it, becomes magnetised and in turn magnetises the second and so on. We get then a chain of magnets, the North Pole

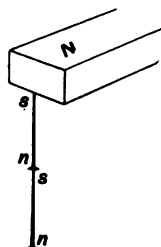


Fig. 120.

of one clinging to the South Pole of the next. The existence of the tuft of filings (Art. 142), is explained in the same way.

The magnetism produced by the presence of a magnet is said to be *induced*. In the cases considered this magnetisation by induction is only *temporary*. To shew this hang a wire nail or key from the pole of a strong magnet and dip the free end into a basin of filings. A bunch clings to it. Now remove the nail from off the magnet. The filings all, or nearly all, fall off.

It is worth while to repeat the experiment, replacing the nail—which is made of soft iron—by a darning needle, razor-blade, or other piece of hard steel. You will notice this difference: when the needle is pulled off the magnet some of the filings will still adhere. The needle is magnetised by

induction in the same way as the soft iron, but the soft iron loses nearly all trace of magnetisation when removed from the neighbourhood of the magnet: the hard steel retains its magnetisation much better. Hard steel can be permanently magnetised; soft iron or steel cannot.

The attraction of a piece of iron by a magnet appears a very simple thing at first sight, but as a matter of fact the attraction is preceded by the magnetisation of the iron. One portion of the iron becomes North and the other South so there are at least four forces to be considered: the two repulsions between the like poles, the two attractions between the unlike. The final approach of a filing to a magnet is due to the fact that two of the unlike poles (N, s), fig. 121, are comparatively close together and that their attraction more than balances the repulsion between the like poles.



Fig. 121.

144. To test the polarity of a magnet. If a piece of magnetised iron is brought close to a compass needle the iron is magnetised by induction from the needle and attracted. Attraction then between a compass needle and a piece of iron is no sure indication of previous magnetisation in the iron.

If, on the other hand, the approach of a piece of iron causes repulsion there must be two like poles near together and the magnetisation of the iron cannot be due to induction. Repulsion is therefore the proper test.

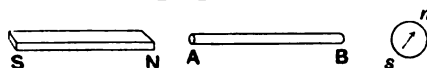


Fig. 122.

Now suppose a magnet, NS , is placed near to a piece of soft iron, AB , as in fig. 122. We know that AB will be magnetised by induction and the experiments above are really sufficient to prove that the end B is North and A is South for (1) iron is on the whole attracted: the two poles nearest together must then be unlike; (2) if the iron is not very soft (the nib experiment of Art. 143) it will retain some magnetism. The magnet NS may be taken away and the ends AB tested with a compass needle. It will be found that B repels the North Pole, A the South Pole of the needle.

If the magnet is placed to point East and West, the end *B* may be tested in position, for the repulsion of the North Pole of a compass needle *ns* is much more marked when the rod is there than when it is removed.

To test the polarity of the near end place the compass and the magnet as in fig. 123. This time the magnet is to point North and South. Then gradually bring up from the left-hand side the rod *AB* into the position shewn. As *AB* approaches the North Pole of the compass *ns* will move away. This indicates that the end *A* is North.

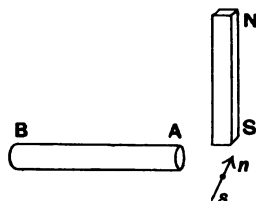


Fig. 123.

145. Induction by the Earth. A piece of iron in the neighbourhood of a magnet becomes itself a magnet. Now the earth is magnetised and therefore all pieces of iron may be expected to shew signs of induced magnetism.

To illustrate this do the following experiment. Take an unmagnetised rod of soft iron. Hold it horizontal in the plane of the meridian and tap it gently with a hammer. Now test both ends by means of a compass needle. You will find that the end which pointed North repels the North end of a compass needle. The rod has therefore been magnetised by induction from the earth. To demagnetise the rod, hold it horizontal pointing East and West and strike it a few times with a hammer.

After it is demagnetised, hold it vertical and tap it again with a hammer. It will become magnetised again: and the North Pole will be at the lower end. If you care to take the trouble you will find that almost any vertical piece of iron—a pillar, rail, gas pipe—is magnetised with a North Pole at the bottom.

146. Lines of Force. Consider the action of a magnet *NS* at a point *P* near to it. Of course if there is no magnetic substance at *P* there will be no action at all. We shall suppose then that we have the North Pole of a small compass needle at *P*. It will be repelled by *N*, attracted by *S*. In the diagram *SP* is about half *NP* so that the attraction to *S*

will be about four times the repulsion from N , for forces are inversely proportional to the squares of the distances between poles. To find the direction of the resultant, mark off a length PA of one unit along NP and a length PB of four units along PS . Complete the parallelogram $PACB$: the diagonal PC gives the direction.

Now if we could get a North Pole at P and it were quite free to move it would begin to travel in the direction PC . In actual practice we cannot get a North Pole all by itself: every magnet has a South Pole as well as a North. We can however test our result roughly by the following experiment.

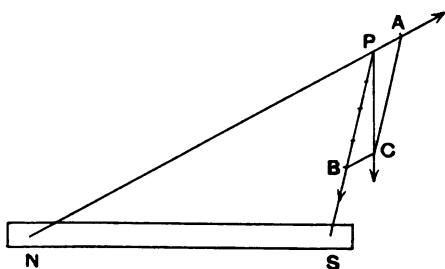


Fig. 124.

Take a long magnetised needle and by thrusting it through a cork arrange it to float vertically in a large trough of water, North Pole upwards. Place a magnet, bar or horse-shoe, nearly level with the surface of the water and close to the floating needle. The needle will move away from the North end of the magnet, trace a curve and finally reach the South end. Note the shape of the path. Repeat the experiment by starting the needle in different places. (Unless the trough is a big one the cork will be pulled to the side by the surface tension of the water.)

The path traced by the needle is called a *line of force*.

This method of tracing lines of force—though very simple and easily understood—is neither exact nor convenient. A much better method is to use a compass needle.

Consider again the magnet NS (fig. 124): the force on a North Pole at P is in the direction PC . This means that if a small compass were brought up to P , its North Pole would

be pulled in the direction PC , its South Pole would be pushed in the opposite direction, CP . The compass would therefore point along the line PC . This is the direction of the 'line of force' passing through P .

147. To trace lines of force. Place a magnet pointing South and North in the middle of a large sheet of paper. Put the compass near the N end of the magnet and make a dot on the paper at each end of the needle. Now move the compass into such a position that the south-seeking end of the needle may be where the north-seeking end was. Make a dot under the north-seeking end, and continue in the same way. Draw a curve through the points thus obtained.

Obtain several other curves in a similar way, starting from different points.

Fig. 125 is the northern half of a drawing obtained in this

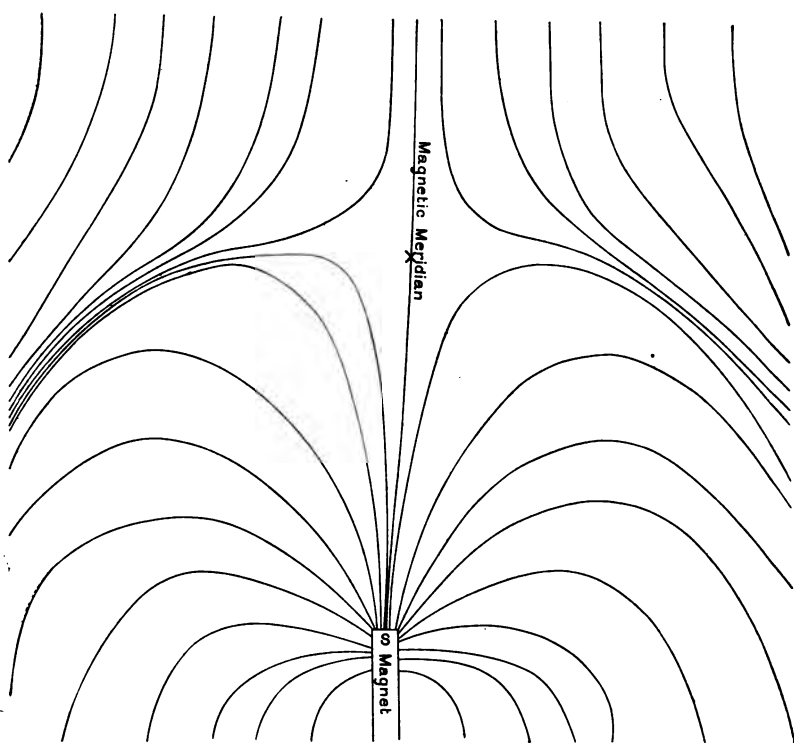


Fig. 125.

way. There is, however, this point to be noticed. The earth acts as a magnet so that the lines of force obtained are due partly to the earth, partly to the magnet.

In the centre of the field the force due to the magnet is much greater than the force due to the earth, and the shape of the lines is only slightly modified by the influence of the earth. On the outlying parts, however, the case is just reversed.

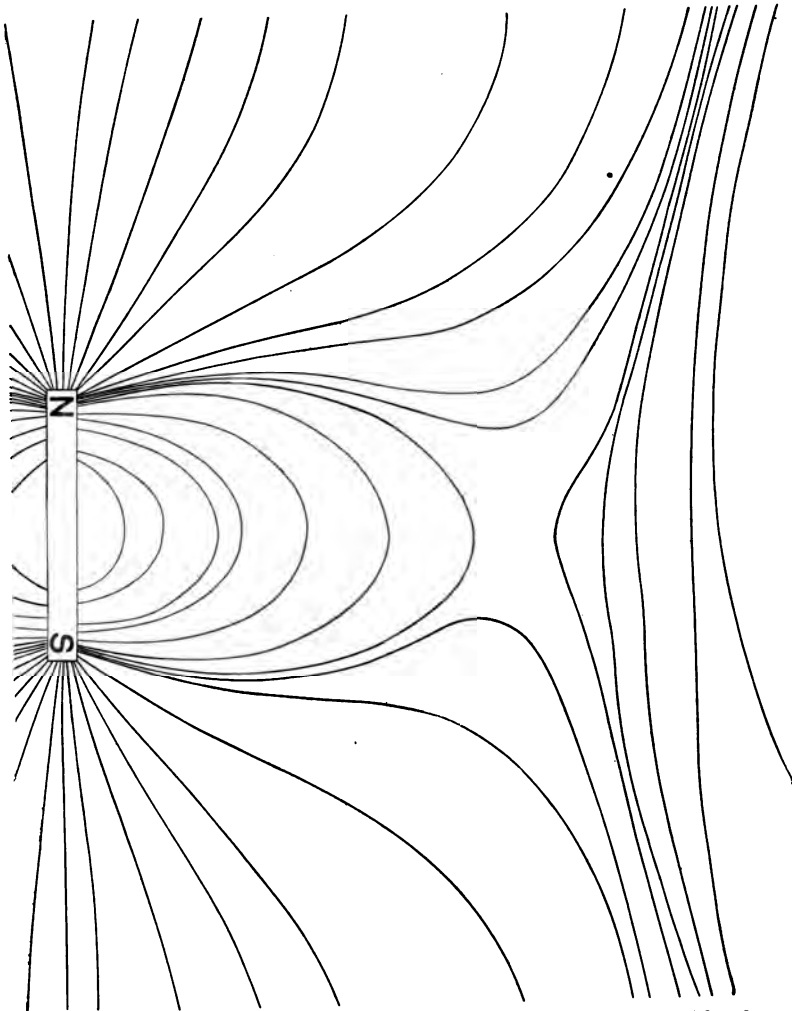


Fig. 126.

10—2

Notice the point marked *X*. At all points in the axis, a compass would point either due North or due South. Near to *S* the magnet's influence will be greater than the earth's so that the compass would point due South: further off the earth's influence will be greater. There must, therefore, be some point at which the two are exactly equal, and where a tiny compass would point indifferently in any direction. Such a point is called a *neutral point*. *X* is a neutral point, and there is a corresponding one further South outside the diagram.

If the magnet is reversed—i.e. points to the North—there are no neutral points on the central line. There are, however, two others, one on either side of the magnet, *v.* fig. 126.

Fig. 127 gives the lines for a magnet pointing at right angles to the meridian.

148. Lines of force traced by filings. If the field which has to be traced is a strong one, iron filings may be used in place of a compass: each little chip of iron is magnetised by induction and its North Pole is ready to join up to the South Pole of its neighbour. In this way chains of filings are formed along the lines of force.

Place a magnet on the bench under the middle of a sheet of white cardboard. Sprinkle the surface with filings shaken from a muslin bag held a foot above. Tap the cardboard gently. Make a sketch of the figures produced.

If you wish to make a permanent record a simple plan is to lay on the top of the cardboard a sheet of paper which has been soaked in hot paraffin wax and allowed to drain and cool. If the filings are scattered over this in the usual way, a little heat will cause them to stick to the paper.

The figures (129 &c.) were obtained by scattering the filings in a dark room on the film side of an ordinary photographic plate: the magnets were below under the glass side. After the plate had been sprinkled and tapped a match was lighted and held a yard above the plate.

Fig. 128 shews the lines due to two magnets with like poles together.

Notice that the outer parts of the field are not very different from those of a single magnet.

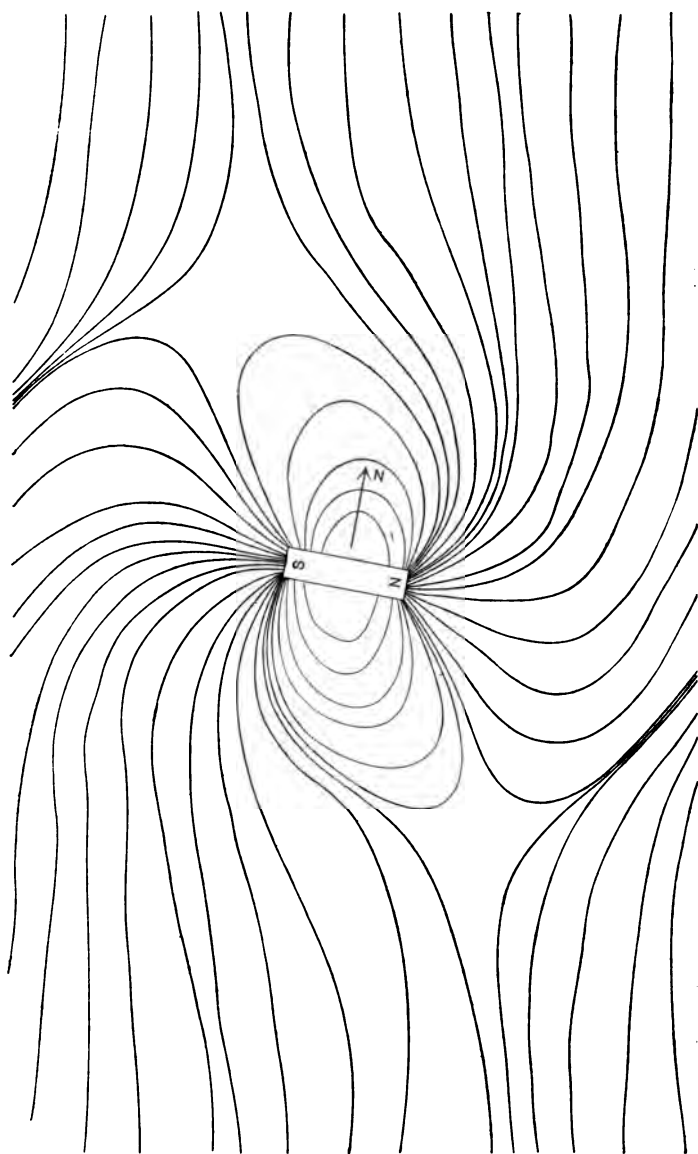


Fig. 127.

Notice also the two neutral points half-way between like poles.

In fig. 129 unlike poles are together: note the neutral point in the centre.

In fig. 130 a bar of soft iron replaces the upper magnet. Note that the field does not seem to extend to the far side of this iron.

In fig. 131 the left-hand half represents half the field due to a horse-shoe magnet without a keeper: the right-hand half is the corresponding figure for the same magnet with the keeper in position. Note how the keeper confines the field. In the first case the field near the pole is so strong that the filings are pulled right away and a bare patch is left.

In fig. 132 we get the field due to three poles, in one case two alike and one unlike; in the other all three alike.

Fig. 133 shews an iron washer between unlike poles. The lines crowd up to the ring and run through the material of it so that none can be detected within the inner circle.

Fig. 134 gives the lines due to a single pole. To obtain this the magnet was vertical and the plate horizontal. Notice the lines crushed to the corners.

Fig. 135 was obtained in the same way but a circular sheet of iron was held between the plate and the magnet.

Fig. 136 is from two magnets placed to form a T.

These figures give a good deal of information about the behaviour of iron in a magnetic field. In the first place we see that the lines of force bend and crowd into the iron. Further they do not seem to cross the iron at all. Thus the bar of iron in fig. 130 'shields' the space beyond it from the action of the magnet. There are no lines of force in the middle of the iron ring, fig. 133.

This screening or shielding effect of iron is very important. A compass needle placed inside a box made of thick soft iron would not be influenced by a magnet outside it. A box of wood or glass or copper would not have a like effect.

149. To magnetise a rod of steel lay it on the bench and draw the pole of a strong magnet from one end *A* to the other end *B* (fig. 137). Lift the magnet off at *B*, replace it at *A* and draw it again over the steel to *B*. Repeat this several times.

If the North end of the magnet is used, the end *A* of the steel will be North, and the end *B* will be South.

It is perhaps more effective to use what is called the method of 'divided touch.' Take two magnets and bring the North Pole of one and the South Pole of the other close together near the middle of the steel. Now draw them apart towards the ends *A*, *B* (fig. 138).

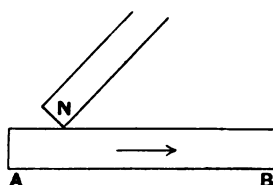


Fig. 137.

If the poles are as shewn, the end *A* will become South, and *B* North. The steel may be laid on two other magnets, *N'*, *S'*, but this is not necessary.

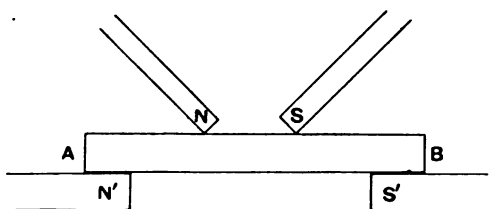


Fig. 138.

150. Consequent poles. Repeat the above operations on another piece of steel, but this time use the two like ends (say N.) of the magnets. The steel will be magnetised to form a double magnet: i.e. it will have like poles (S.) at the ends, and double or consequent poles (N.) at the centre.

151. Molecular Theory of Magnetisation. If a glass tube is loosely packed with steel filings and treated in the same way as the steel bar in article 149, the filings can be seen to rise up on end and turn over as the magnetic pole passes across them. Further, the rough surface is smoothed out and the filings have joined up in line. If the condition of the tube is tested with a compass needle, it will be found to be magnetised to some extent. The slightest shake, however, seems to destroy this orderly arrangement and with it the magnetisation.

Now a piece of soft iron seems to behave in much the same way as the tube of filings. It also can be magnetised

but a shake destroys its magnetisation. We might imagine that the iron is made up of small particles, that each part is itself a magnet. In the ordinary condition these particles are arranged haphazard and point in different directions. The presence of a strong magnet might make many of these little 'molecular magnets' turn round and face in a definite direction.

Fig. 139 is an attempt to illustrate this imagined arrangement. Here the N. Pole of one magnet comes close to



Fig. 139.

the S. Pole of its neighbour so that the two practically neutralise. There are, however, a set of N. Poles at one end and S. Poles at the other which have not been so eliminated : such a collection of little magnets would therefore act as one big magnet. A shake would disturb the arrangement and so demagnetise the iron.

Such a theory accounts for the properties of soft iron, so far considered. Hard steel we may suppose differs from soft in that, though similarly constituted, the 'molecular magnets' cannot arrange themselves so readily: when arranged, however, they are more firmly fixed. This will explain why hard steel is less readily magnetised than soft iron: but when magnetised its magnetism is more permanent. Soft iron or steel can only have temporary magnetism; hard iron or steel can retain its magnetism to some extent, but every shake or knock tends to destroy it.

Take a knitting needle, notch it with a file in the middle. Then magnetise by any method and break it in two at the notch. Test each part with a compass. You will find you have two magnets, each having a North and a South Pole. If you care to break the halves again, you will get four complete magnets; and so on till you get to quite small pieces, all of which are magnets.

152. Demagnetisation. It will be convenient to use a needle. Test its condition by a compass and then hold it in a Bunsen flame till quite red hot. Now bring it close to the needle, you will find there is no action: when red hot, iron is non-magnetic. Now remove the needle from the compass, heat it up again and drop it into water to cool it quickly.

Test it again by the compass and you will find that, though magnetic, it is no longer magnetised.

If the steel had been allowed to cool in a strong magnetic field, it would have regained its magnetism. To do this, place it while hot between the poles of a strong horse-shoe magnet.

Heating and shaking destroy the magnetism of a piece of steel: but even if not subjected to these actions no magnet is really permanent: it can demagnetise itself. Consider what would be the behaviour of a little compass needle placed on the top of a bar magnet, as in fig. 140; it would point towards the South Pole of the magnet.

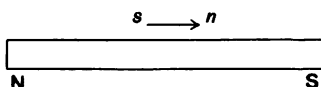


Fig. 140.

But on the theory we have just described the little 'molecular' magnets point in the opposite direction. They must therefore be under the action of forces which tend to reverse their direction. This reversal would mean demagnetisation.

This tendency to reverse can be eliminated or decreased by putting near *S* a North Pole, near *N* a South Pole. Two magnets then placed side by side, as in fig. 129, will retain this magnetism much better than if kept separately, especially if their ends are joined by soft iron bars or 'keepers.'

The purpose of the keeper with which a horse-shoe magnet should always be provided is now apparent. It joins the poles together and becomes magnetised by induction: so that each pole is closely joined to an opposite one.

153. Ewing's experiment. If a large number of compass needles be taken and grouped together, by mounting them on a board, it is found that they do not all point toward the North but that one influences another so much that their arrangement appears haphazard. Prof. Ewing took a collection of needles, such as this, and put it in a magnetic field the strength of which was gradually increased. While the field was weak there was no marked effect: a few needles only changed their direction, but on the field being made stronger more and more needles pointed in the direction of the magnetic force until finally they all did so, a great number turning round all at the same time.

As the strength of the field was gradually decreased, a few needles only lost their alignment; most of them retained it until the external magnetic force was removed. A slight shake was then sufficient to cause them to take up positions and groupings similar to those they held originally.

This experiment illustrates almost exactly the behaviour of a piece of soft iron and is further evidence in support of the molecular theory.

154. Magnetic substances. The only substances capable of being permanently magnetised are iron and some of its compounds such as steel and the oxide of iron which is sometimes found as a natural magnet. But though no other substances can be turned into magnets, yet many are influenced by a magnet to some slight extent. Such substances are called magnetic. Excluding iron, the most notable magnetic substances are nickel and bismuth. The former is attracted by a magnet and can be tested by a compass needle in the ordinary way, though it is better to hang up a bar of the metal between the poles of a powerful magnet.

The behaviour of bismuth is quite different. When treated in the same way as the nickel, instead of being attracted to one of the poles it turns at right angles to the line joining the poles and is repelled from the strong part of the field.

Iron, nickel and cobalt are attracted by a magnet and set themselves along (or **parallel**) to the lines of force: they are called **paramagnetic** substances: bismuth and antimony are repelled by a magnet and set themselves across the lines of force. They are called **diamagnetic**.

In a really powerful field it can be shewn that almost all substances are magnetic to some extent: thus wood, paper, &c. are attracted but only very slightly. It is remarkable how great a difference there is between the magnetic properties of iron and of all other substances.

155. Temperature effect. The magnetic properties of iron are dependent on temperature. The higher the temperature the more strongly magnetic, provided that a certain limit is not passed. Beyond this critical temperature, about 700°C. or 800°C. , iron ceases to be magnetic at all. In other words the susceptibility vanishes and the

permeability falls to unity (Art. 175). The temperature is apparently one at which the structure of the iron is in some way altered: for if a piece of the iron is allowed to cool down to it after being red hot it will suddenly emit a distinct but transient glow accompanied by a *rise* in temperature. For this reason the temperature is called that of *recalcescence*. If a knitting needle is heated in a dark room in a Bunsen flame and then allowed to cool, this phenomenon can often be observed. The exact temperature of recalcescence is dependent on the kind of iron or steel. So also is the critical temperature.

The permeability of iron is not a definite quantity even for the same specimen: it is dependent on the strength of field. For very strong fields the above results are not true: for it is found that permeability then decreases with rise in temperature: in any case however iron ceases to be magnetic at the critical temperature.

EXAMPLES

1. How could you conveniently arrange a horse-shoe magnet so that it would act as a compass needle?

2. A magnet pointing S. and N. lies on a table. At several different distances both E. and N. of the centre of this are placed small compasses. Draw a diagram shewing how these compasses would point, and indicate which would reverse if the magnet were to be reversed.

3. What is meant by magnetic matter and magnetised matter? How would you distinguish between them?

4. Both ends of a steel bar attract the N.P. of a needle. What is the magnetic state of the bar?

5. How could you, by means of magnetic experiments, distinguish between bars of brass, soft iron, hard steel, and a bar magnet?

6. One end of a bar of steel is brought near the N.P. of a horizontally suspended magnet. At first there appears to be slight repulsion, but there is attraction when the distance between the two is decreased. Explain this.

7. A bar of iron is strongly magnetised. How could you shew that the outside parts only are affected?

8. Why is a magnet often made lamellar?

9. 'A magnet attracts iron; but when we analyse the effect we learn that the metal is not only attracted but repelled, the final approach to the magnet being due to the difference of two unequal and opposing forces.' Tyndall.

Explain and criticise this statement.

10. How would you magnetise a bar of steel so that it may have consequent S. Poles at the centre?

11. Two compass needles are placed on a table, one a considerable distance S.W. of the other. What influence will they have on each other?

12. A compass needle is placed a long way due S. of a strong magnet, which lies in the magnetic meridian. Describe the behaviour of the needle as it is carried up to, over, and beyond the magnet.

13. An iron ball is held over a pole of a horse-shoe magnet. Will the attraction exerted on the ball be altered if the poles of the magnet are connected by a soft iron keeper, and, if so, in what way and why?

14. A magnet is placed horizontally in the magnetic meridian due south of a compass needle. How will its action on the latter be affected if (1) a plate of soft iron is interposed between the two, (2) a rod of soft iron is placed along the line which joins their centres? Give reasons.

15. A strong magnet is placed with its centre due E. of a compass needle. If the magnet is horizontal and points N. and S., and is then turned round in the direction N., E., S., W., N., about a vertical axis, what will be the changes in the indications of the compass?

16. A magnet with its north-seeking pole E., and its south-seeking pole W., is placed with its centre the same distance N., E., S., and W., successively, of a small compass needle on a table. Give four figures shewing the nature of the deflexion of the compass needle in each case.

17. A piece of soft iron wire is held in a vertical position with its lower end near the North Pole of a compass needle, which is feebly attracted. The wire is then heated bright red by a flame, when it is observed that the attraction ceases. As the wire cools the needle is observed to be repelled.

Explain these phenomena.

18. How could you (1) magnetise a ring of iron, (2) test the magnetic condition of such a ring?

19. A magnet is placed near a compass needle. In what ways could we, without removing the magnet, get rid of nearly all its action on the needle?

20. A rod of soft iron is placed horizontal and perpendicular to the magnetic meridian so that its axis produced passes through the centre of a compass needle. How is the needle affected by the iron, and how is the effect altered when the end of the rod remote from the needle is gradually raised until the rod is vertical?

21. How would you place a rod of soft iron for it (1) not to be magnetised, (2) to be magnetised as much as possible along its length by the earth's inductive action? Give your reasons.

22. *ABCDE* is a pentagon. A large magnet lies along *AB*. Small compass needles are placed at the corners. Shew in a diagram the positions they will assume.

23. It is desired to have a compass which will oscillate as quickly as possible. What sort of magnet would you use, and how would you arrange it?

CHAPTER XVII

TERRESTRIAL MAGNETISM

156. To determine the magnetic axis of a disc and the meridian at any place. If a magnetised disc is hung up by a hook from the centre it will come to rest with its magnetic axis pointing North and South. To determine this axis, rule any diameter on the disc and mark the position this line takes up when the disc comes to rest. This may conveniently be done by holding the disc over a horizontal drawing board and marking the positions of the ends of the diameter by means of pins. Now reverse the disc, so that the upper side comes underneath. When the oscillations have ceased, again mark the position of the diameter. The line bisecting the angle between the two directions so obtained gives the true compass North. That this is so is quite easy to understand, for turning the disc upside down cannot alter the direction of the magnetic axis which must be N. and S. so that any other diameter must be inclined as much West of it after reversing as it was East of it before.

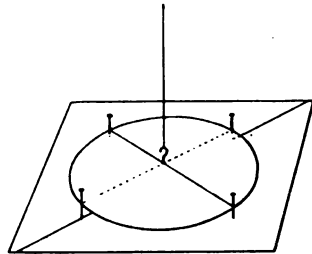


Fig. 141.

157. The Meridians. At most places on the earth a compass needle points in a northerly direction; not exactly North, but a few degrees East or West of North. We do not know the reason for this: in fact we do not know why it should point in any particular direction at all.

To find true North—or geographical North—we may use the sun or the stars: for these are due South of us when highest in the sky. It is quite worth while to do the following experiment. Set up a stick vertically in the ground and take observations of the length of its shadow. When this is least, the Sun is due South and the shadow is pointing North and South. The time of this will always be between 11.40 and 12.20 by mean (clock) time. You cannot get a very accurate North and South in this way, for the shadow is too indistinct to be easily measured, but you will quite readily see that the line is not in the direction in which the compass points.

We define the plane of the *geographical meridian* at any place as the plane passing through that place and also through the axis of the earth. It is a vertical plane, true North and South.

The plane of the *magnetic meridian* at any place is defined as the vertical plane passing through the axis of a compass needle. The angle between the two meridians is called the *Declination*. Perhaps it is simpler to say that the declination is the angle between true North and compass North.

158. The Declinometer is an instrument for the accurate determination of the magnetic North and South. A diagrammatic sketch of this is shewn in fig. 142. It consists of a telescope *TT* mounted on a bar. The bar is fixed to a table capable of revolving on a circular horizontal stand. This stand is graduated on the edge and the table carries a vernier so that we can read azimuth changes, i.e. the angle through which we turn round the vertical axis. Over the centre of the table hangs the magnet *ns*, supported by a long silk fibre, free from torsion. This magnet is a steel tube magnetised along its length. The end remote from the telescope (*n*) carries a glass scale ruled vertically; the other end (*s*) is fitted with a lens, the focal length of which is equal to the length of the magnet; so that the image of the scale can be seen by the telescope. Now if the observer brings the image of the middle of the scale on the crosswires of the telescope, he knows that he is looking along the geometrical axis of the magnet and can find the azimuth

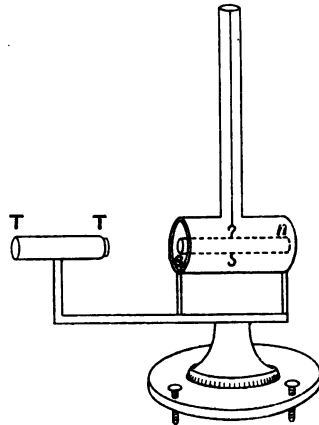


Fig. 142.

by reading the scale and vernier. If before he put the magnet in position he had pointed the telescope on an object due (geographical) North and had taken the scale reading, the difference between the two readings would give the declination, provided that the magnetic axis of the tube coincided exactly with its geometrical axis. In actual practice this condition is not quite fulfilled. To correct for the error the tube is turned upside down and a new azimuth reading is taken. The mean of the two readings gives the true direction of the magnetic axis (Art. 156).

159. The declination varies at different places and at different times. Thus in London in 1580 the declination was about 11 degrees E., and in 1657 it was zero, the compass pointing true North. After this date it gradually rose to its greatest value of $24\frac{1}{2}$ degrees W. in 1820, when it began to decrease till it is now about 16 degrees W.

In addition to this *secular* change in the declination, there are regular annual and diurnal changes, as well as irregular variations connected with solar disturbances.

160. Inclination. A compass needle—like a weather cock—is mounted on a vertical axis and can only point in a horizontal direction. It does not shew the actual direction of the earth's magnetic force, any more than a weather cock which is uninfluenced by an ascending or descending current, shews the actual direction of a breeze. A magnetic needle, if it is to shew the true direction of the earth's field, ought to be mounted at its centre of gravity and free to point in any direction. A very rough approach to this mounting may be effected by taking a long knitting needle, cutting a deep notch in the middle and tying it up by a single fibre of silk. One end will probably be heavier than the other: balance it by adding a little rider of cork and adjust till the needle is horizontal. Now magnetise the needle and hang it up again. It will no longer be horizontal but the North end will dip down considerably. The angle which such a needle would make with the horizontal, if suspended freely at its centre of gravity, is called the *inclination* or *dip*.

Now the vertical plane passing through the axis of the needle is the plane of the meridian: it follows therefore that to find the angle of dip it is not necessary for the needle to be quite freely suspended: it would be sufficient if it were free to move in the plane of the meridian. All accurate dip

instruments have this freedom only: they consist of a magnetised needle mounted on an axle. When placed on its bearings this axle must be horizontal and point East and West (i.e. at right angles to the plane of the meridian). The needle is then free to swing in the meridian and to point in the direction of the earth's field.

161. To find the inclination by the dip circle.

Level the instrument. Turn the circle about a vertical axis until the needle is vertical. Its plane is now E. and W. Turn it round through a right angle so that its plane may be N. and S. (Unless the instrument is a delicate one, it will be better to obtain the plane of the meridian by the use of a compass.) The reading may now be taken, but it is liable to three important errors:

(1) *Centre error*, due to the centre of the needle not being at the centre of the graduated circle. Correct for this by reading both ends.

(2) The magnetic axis may be inclined to the geometric axis. Correct for this by turning the needle over, and again read both ends. (Cf. Art. 156.)

(3) *Centre of gravity error*: One end of the needle may be heavier than the other. Correct for this by re-magnetising in the opposite direction.

Hence we must (1) read both ends, (2) turn the needle over and read both ends, (3) reverse the magnetism and repeat (1) and (2).

Shew with diagrams how the above methods supply the desired corrections.

162. Isoclinic and Isogonic lines. As a general rule a magnetic needle suspended at its centre of mass will, at places North of the equator, dip with the N. pole down. The nearer we get to the equator the more nearly horizontal will the magnet be. After crossing to the southern hemisphere the S. pole dips down and the N. is tilted up. Roughly, the needle behaves as if the earth's field were due to a small strong magnet placed at the centre of the earth with its S. end pointing towards the so-called North Pole. The axis of this magnet would not coincide with the axis of rotation of

the earth, but would be inclined to it at a small angle and point towards Boothia Felix. To account for the secular changes we must suppose that this magnet revolves slowly round once in 960 years, always making an angle of about 15 degrees to the axis of the earth.

If on a map of the world lines are drawn joining up all places at which the dip has any particular value, the curves so obtained are called *isoclinic* lines. They roughly conform to the lines of latitude.

Lines which join up places of equal declination are termed *isogonic*. There are two isogonic lines on which the declination is zero. One of these runs in a large oval round Japan as centre; the other starting from Boothia Felix runs South-Easterly, crosses Brazil and passing right across the Antarctic cuts off a corner of West Australia and takes a North-West course over the Indian Ocean and Europe up into the Arctic to Boothia Felix again.

These two lines of zero declination are called the *agonic* lines.

EXAMPLES

1. How could you determine the magnetic axis of a plate of steel?
2. A compass on board ship is affected by the iron on the ship. How can this effect be decreased and allowed for?
3. What is the direction of the lines of force due to the earth at London? What experiments would you do to find it?
4. How can a dip needle be used to find the meridian?
5. What would be the behaviour of a dip needle carried along a meridian from N. to S. pole?
6. If a bar of soft iron were freely suspended at its centre of mass, would it remain at rest in any position?
7. A long soft iron rod is held vertically above the centre of (1) a compass needle, (2) a dipping needle. Explain what effect, if any, it has on the needles.
8. In what positions can a compass be placed if it is to be uninfluenced by a vertical magnet?
9. A tall iron mast is situated a little in front of the compass in a wooden ship. Explain the nature of the compass error when the ship is sailing in an easterly direction (1) in the northern, (2) in the southern hemisphere.

10. A small weight is attached to the upper end of a dip needle, whereby the inclination of the needle to the horizontal plane is diminished by one-half. Draw a diagram shewing the directions of the forces which act upon the needle, and state the conditions of equilibrium.

11. A road in the northern hemisphere runs due north and south. At one point an insulated conductor passes beneath it, in which an electric current flows from east to west. How will the indications of a dip needle be affected at points near to the conductor?

12. In adjusting a dip-circle the vertical circle is turned through an angle which differs from a right angle by an angle θ owing to an error in the graduations of the horizontal circle. Prove that the angle of dip as found from this instrument is $\tan^{-1}(\tan \delta \sec \theta)$, where δ is the true angle of dip.

QUANTITATIVE MAGNETISM

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Then, if n be the strength of either pole of the magnet and m that of the pole P ,

$$\text{the force on } P \text{ due to } S = \frac{mn}{(d-l)^2},$$

$$\text{the force on } P \text{ due to } N = \frac{mn}{(d+l)^2}.$$

These forces are in opposite directions, and their resultant will therefore be measured by their difference.

Hence the force due to magnet

$$\begin{aligned} &= \frac{mn}{(d-l)^2} - \frac{mn}{(d+l)^2} \\ &= \frac{4mnl d}{(d^2 - l^2)^2} \\ &= \frac{4mnl d}{d^4} \text{ if we neglect } l^2 \\ &= \frac{2m\mathbf{M}}{d^3} \text{ if } \mathbf{M} = 2ln. \end{aligned}$$

We shall make a very small mistake in neglecting l^2 if l is small compared with d , i.e., if the magnet is short.

Thus the force exerted by a magnet on a magnetic pole in this position is proportional to the cube of the distance.

The quantity $\mathbf{M}$, which is equal to the product of the strength and the distance between the poles, is called the **moment** of the magnet.

The ratio of the moment to the volume is called the **intensity of magnetisation**.

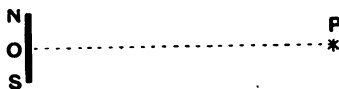


Fig. 144.

- (2) In this case, viz. when OP bisects NS at right angles,

$$\text{the force due to } N \text{ on } P = \frac{mn}{NP^2},$$

$$\text{the force due to } S \text{ on } P = \frac{mn}{SP^2} = \frac{mn}{NP^2}.$$

By the parallelogram of forces we find that the resultant of these forces is perpendicular to OP , and

$$\begin{aligned}
 &= \frac{2mn}{NP^2} \sin NPO \\
 &= \frac{2mn}{NP^2} \cdot \frac{ON}{NP} \\
 &= \frac{2mnl}{NP^3} \\
 &= \frac{m\mathbf{M}}{OP^3},
 \end{aligned}$$

if NS is so short that NP^3 and OP^3 may be considered equal,

$$= \frac{\mathbf{M}m}{d^3}.$$

165. Couple acting on a magnet in a uniform field.

Suppose a magnet, of a pole strength m , and length l , is placed in a uniform field of strength $\mathbf{H}$ and inclined to it at an angle θ .

Then the force acting on each pole will be $\mathbf{H}m$.

The distance between the lines of action of these forces is $l \sin \theta$.

Hence the moment of the couple acting on the magnet

$$= \mathbf{H}ml \sin \theta = \mathbf{H}\mathbf{M} \sin \theta,$$

where $\mathbf{M}$ is the moment of the magnet.

Hence the moment of a magnet is measured by the moment of the couple exerted on it when placed in and at right angles to a field of unit strength.

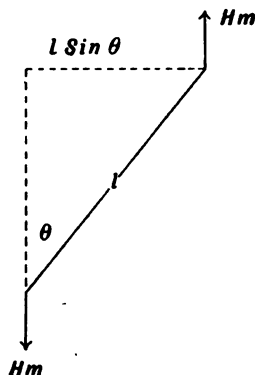


Fig. 145.

166. Deflexion of a compass by a magnet. Suppose a magnet pointing East and West is due West of a point P in the earth's field. Then the resultant horizontal field at P is due to the two components :

(1) the force due to the earth: this is due North and is indicated by $\mathbf{H}$.

(2) the force due to the magnet. If the magnet is short compared with its distance from P , this force is $\frac{2M}{d^3}$. (Art. 164.)

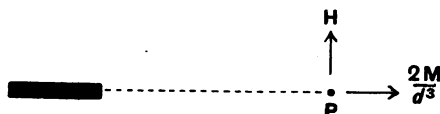


Fig. 146.

If this resultant makes an angle A with the meridian, then (Art. 8)

$$\frac{2M}{d^3} / H = \tan A, \text{ or } \frac{M}{H} = \frac{d^3}{2} \tan A,$$

but the direction of this resultant is the direction in which a small compass needle situated at P would point. Hence the deflexion of a compass needle from the meridian by a magnet placed "*end on*" to it is given by the above equation.

If the magnet, still pointing East and West, had been placed a distance d due North or South of the centre of the compass, it would have exerted a force $\frac{M}{d^3}$ and the deflexion B produced would have been given by the relation

$$\frac{M}{H} = d^3 \tan B.$$

This position is known as "*broadside on*."

An instrument used to investigate the deflexion of one magnet by another is called a magnetometer. For exact work the deflected magnet is usually very small and suspended as in a mirror galvanometer. A simpler instrument often used consists of a small compass mounted at the centre of a graduated card. The deflexion of the needle is read by a long pointer fixed to it. Two arms, each about half a metre long, are attached to the compass box.

167. The Magnetometer. Place the magnetometer with the arms E. and W., with the pointer at the zero. Place the magnet "*end on*" on the E. arm and note the reading at each end of the pointer.

Calculate the distance (d) between the centres from readings obtained by measuring to each end of the magnet from the centre of the scale.

Now reverse the magnet, keeping the centre in the same position. Again read the deflexion.

Place the magnet on the W. arm, at the same distance from the centre as before and so obtain another set of results. You will thus obtain eight readings of the deflexion. Calculate their mean (A).

Take several further readings for different distances.

Tabulate the results and plot a curve connecting the cube of the distance (ordinate) with the cotangent of the deflexion (abscissa).

If the relation $\frac{M}{H} = \frac{d^3}{2} \tan A$ were exactly true, we should get a straight line as the result: the variation shews to what extent the inverse cube law is actually followed.

Repeat the experiment in the "*broadside-on*" position.

Plot both curves on the same sheet. Shew that for the same deflexion the cube of the distance in one case is double that in the other.

168. Gauss's proof of the law of the inverse square.

The preceding experiment verifies the conclusion of Art. 164, that the deflecting couple exerted by a magnet placed "end on" to another is twice as great as when it is "broadside on."

Gauss shewed that this constitutes a proof of the law of the inverse square; for let us suppose that the force between two magnetic poles varies inversely as the p th power of the distance.

Consider the intensity of the field due to a short magnet, NS , centre O , at distant points C and D : C being in the line NS , and DO perpendicular to it.

Let $OC = OD = d$, $NS = 2l$, strength of pole = m , $-m$, $2lm = M$.

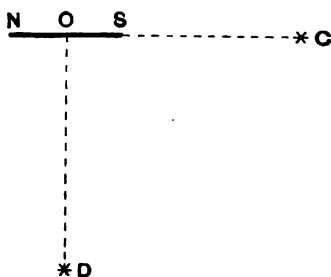


Fig. 147.

(1) Force at C

$$= \frac{m}{SC^p} - \frac{m}{NC^p} = m \left\{ \frac{1}{(d-l)^p} - \frac{1}{(d+l)^p} \right\} = \frac{p \cdot \mathbf{M}}{d^{p+1}} \text{ (approx.)},$$

(2) Force at D

$$= \frac{2m}{ND^p} \sin NDO = 2 \frac{ml}{ND^{p+1}} = \frac{\mathbf{M}}{d^{p+1}} \text{ (approx.)}.$$

From these results we see that if a *small* magnet is placed "end on" to a small compass needle it would exercise a deflecting couple p times as great as it would if placed "broad-side on."

To find the value of p .

Place the magnet, pointing E. and W., a considerable distance, d , due E. of a small compass. The latter must be so arranged that its deflexion ($\mathbf{A}$) may be read by means of a beam of light reflected from a mirror attached to it.

From the results obtained above we must have

$$\frac{p\mathbf{M}}{d^{p+1}} = \mathbf{H} \tan \mathbf{A}.$$

Next place the magnet due S. of the compass, and again find the deflexion ($\mathbf{B}$). We shall get

$$\frac{\mathbf{M}}{d^{p+1}} = \mathbf{H} \tan \mathbf{B}.$$

The ratio of these two tangents gives the value of p .

When experiments are carefully made in this way the value of the ratio is found to be exactly 2. Hence the law of the inverse *square* is experimentally proved.

We deduce from the results (1) and (2) above that if a magnet is placed in either of the usual positions on a magnetometer, then the tangent of the deflexion is inversely proportional to the $(p+1)$ th power of the distance. By actual experiment we find that it varies as the inverse cube; hence $p+1=3$, and as before we get the value 2 for p .

169. Comparison of magnetic moments of two magnets.

Method (1).

Place the magnetometer with the arms East and West with the pointer at zero. Place one of the magnets on an arm of the magnetometer near enough to the needle to cause a considerable deflexion.

Measure this deflexion and the distance as in Art. 167.

Repeat the experiment with the other magnet, placing it in the same position as the first, and again read the deflexion.

Compare the moments of the magnets by means of the relation of Art. 167.

Take other readings by varying the distances of the magnets from the centre.

Method (2). "No deflexion" method.

Place the two magnets one on each arm of the magnetometer, and so arrange their distances that the needle may not be deflected. In this case the moments will be proportional to the cubes of the distances of the centres from the needle.

Verify by this means the result of your first experiment.

170. Time of oscillation of a magnet. When a magnet is suspended with its axis horizontal so that it can turn freely about a vertical axis, it will oscillate from side to side if disturbed from its position of rest. The time of oscillation depends upon

- (1) the horizontal component of the field (H),
- (2) the moment of the magnet (M),
- (3) the moment of inertia of the magnet about the axis of suspension (K). This can be determined from a knowledge of the shape, size, and mass of the magnet. For an ordinary bar magnet

$$K = m \frac{l^2 + b^2}{12},$$

where l = length, b = breadth (horizontal), and m = mass.

The time of oscillation is independent of the amplitude provided that this is small and is given by the relation

$$T = 2\pi \sqrt{\frac{K}{MH}}.$$

From this result it follows that

- (1) with the same magnet the strength of the field in which it swings is proportional to the square of the number of oscillations per minute. This gives an experimental method of comparing the strengths of different fields.

(2) with magnets of the same mass and dimensions the magnetic moments are proportional to the square of the number of swings per minute when they oscillate in the same field.

171. Comparison of magnetic moments by the method of oscillation. You require two magnets of same shape and mass, bell jar, silk, watch.

Take one of the magnets and support it in a paper stirrup. This is to be hung up by a fibre of silk tied to a piece of wire laid across the top of the bell jar. The magnet must be evenly balanced so that it can swing freely in a horizontal plane.

Place a small mark on the glass opposite one end of the magnet when at rest. Now let the magnet oscillate through a *small* angle. Find the time required for (say) 20 swings, beginning to count as the magnet passes through the equilibrium position. Deduce the time for one oscillation (t_1). Find in a similar way the time for the second magnet (t_2). Apply the result of Art. 170, and so compare the moments of the magnets.

172. To find M and H . The results of Arts. 166, 170 enable us to calculate the moment of the magnet and the strength of the earth's field.

For suppose we have a magnet and place it due West of and "end on" to a compass: the deflexion it produces is given by the relation

$$\frac{M}{H} = \frac{d^3}{2} \tan A \dots\dots\dots(1).$$

If the magnet is suspended as in Art. 171, its time of oscillation is given by the relation

$$T = 2\pi \sqrt{\frac{K}{MH}},$$

i.e. $MH = \frac{4\pi^2 K}{T^2} \dots\dots\dots(2).$

If we multiply (1) and (2) we get an expression for M^2 .

If we divide (2) by (1) we get an expression for H^2 .

To obtain results which are reasonably correct considerable care must be taken. Some form of mirror magnetometer

will probably be necessary, for with the ordinary form of instrument the magnet has to come close to the compass needle to produce a deflexion large enough to be read with accuracy. When this is the case, the formula (1) ceases to be true.

173. Determination of the earth's field at any place. For a complete knowledge of the earth's field at any place we require to know

(a) its direction ;

(b) its intensity.

(a) The direction of the field is generally given in terms of the dip and the declination. The former is determined by the dip needle as in Art. 161 and the latter by the declination compass.

(b) Let $\mathbf{I}$ denote the total intensity of the earth's field. Resolve this into two components, one horizontal, $\mathbf{H}$, the other vertical, $\mathbf{V}$.

Hence if δ is the angle of dip we must have

$$\mathbf{H} = \mathbf{I} \cos \delta, \quad \mathbf{V} = \mathbf{I} \sin \delta.$$

These quantities vary from place to place on the earth's surface.

In London the values are approximately

$$\mathbf{I} = \cdot 47 \text{ dyne}, \quad \mathbf{H} = \cdot 185 \text{ dyne}, \quad \mathbf{V} = \cdot 44 \text{ dyne}, \quad \delta = 68^\circ.$$

EXAMPLES

1. The repulsion between two exactly similar poles is 81 dynes. If they are 4 cm. apart, what is the strength of each ?
2. What is the force between two poles of 100 and 250 placed one metre apart ?
3. The force between two poles placed 10 cm. apart is equal to the weight of 1 gm. What will be the force between them when they are placed (1) 3 cm. nearer together, (2) 4 cm. further apart ?
4. A north pole, strength 75, and a south pole of strength 125, are placed half a metre apart. At what point might a pole be placed not to be influenced by them ? What will be the strength of field half-way between them ?

5. If two magnetised needles were suspended vertically, could they form an astatic pair?

6. A knitting needle balances at the centre when unmagnetised, but two millimetres from the centre when magnetised. Its length is 25 cm., its weight 2 grm., and $V=0.44$ gauss. Find the pole strength.

7. A uniformly magnetised magnet 10 cm. long and 4 sq. cm. cross section has a moment of 1000 c.g.s. units. Calculate the strength of either pole, and the intensity of magnetisation of the steel.

8. Two magnets, each of pole strength 50 and weighing 40 grm., are placed parallel to one another and one cm. apart, the one lying on the table, the other suspended from the underside of the pan of a balance. What will be the apparent weight of the suspended magnet?

9. A horizontally suspended magnetic needle makes 13 vibrations per minute when no magnets are near. When a bar magnet is placed north of the needle with its axis in the magnetic meridian and its N. pole towards the needle, the latter makes 7 vibrations per minute. How many will it make when the bar is similarly placed at the same distance south of the needle, its N. pole being again directed towards the needle?

10. Calculate the magnetic force at a point on the axis of a bar magnet at 100 cm. distance from the centre of the magnet, the strength of each pole being 100 and the length of the magnet being 4 cm.

11. The magnetic force, due to a magnet of length 10 cm. at a point 10 cm. from its centre in the prolongation of its axis, is calculated on the assumption that this force varies inversely as the cube of the distance from the centre. Find the error % thereby introduced.

12. $ABCD$ is a square, length of side 12 cm. At the centre are two small magnets, each of moment 8, placed one along each diagonal. What will be the intensity at A and at the middle point of AB ?

13. A suspended magnet, free to turn about a vertical axis, makes 10 oscillations per minute at a place where the horizontal component of the earth's magnetic force is 0.18 dyne per unit pole. At another place where the angle of dip is 45° the magnet makes 12 oscillations per minute. Calculate the value of the earth's total magnetic force at this latter place.

14. A magnet 2 cm. long with poles of 20 units is placed in a field of intensity 2 gauss, making an angle of 30° with the lines of force. Find the couple tending to turn it.

15. A small compass needle is observed to be in neutral equilibrium when laid on a table at a distance of 20 cm. due north (magnetic) of the south pole on a bar magnet lying on the table with its axis in the magnetic meridian. If the horizontal component of the earth's field is 0.180 c.g.s. and the distance between the poles of the magnet is 10 cm., find the magnetic moment of the magnet.

16. Two short bar magnets are placed with their axes east and west, and their centres at distances of 15 and 20 cm. respectively east and west of a small compass needle. If the needle shews no deflexion, find the ratio of the moments of the magnets, and explain the principle of the experiment.

17. Two short bar magnets, the moments of which are 108 and 192 respectively, are placed along two lines drawn on the table at right angles to each other. Find the intensity of the magnetic field due to the two magnets at the point of intersection of the lines, the centres of the magnets being respectively 30 and 40 cm. from this point.

18. A magnet of moment 75 pointing E. and W. is placed 15 cm. due S. of a compass needle. What is the deflexion produced if the horizontal component of the earth's field is 0.25?

19. At Kew $H=0.18$, and in Iceland $H=0.12$. If a magnet placed on a magnetometer produces a deflexion of 30° at Kew, what deflexion will it produce if placed in the same position in Iceland?

20. Two magnets are placed in position "A" on the magnetometer and produce no deflexion. If their distances from the centre are 25 cm. and 35 cm., compare their moments.

21. A magnet of moment 250 is placed 75 cm. "end on" and due west of a small compass needle. A second magnet of moment 175 is placed due S. of this compass. How far off must it be that the needle may not be deflected?

22. Two small magnets of moment M form a T of which the horizontal arm lies E. and W. Find the force exerted on a pole of strength m placed at a distance d due south.

23. Being given a compass needle and a bar magnet, shew how you would determine the moment of the magnet assuming H to be 0.18.

24. A small magnet is placed parallel to and due west of a compass needle one metre away from it, and the time of oscillation of the needle is decreased from 2.5 seconds to 1.25 seconds. What is the moment of the magnet? What difference would there have been if the magnet had been (1) reversed, (2) placed due S.? ($H=0.2$.)

25. A magnet 2 cm. long, with poles of 20 c.g.s. units strength, is placed in a magnetic field of intensity 0.2 c.g.s. units, making an angle of 30° with the lines of force. Find the couple tending to turn it.

26. What is the intensity of field at a point 12 cm. from the centre of a small magnet of moment 172.8, the point being in line with the axis of the magnet? If a compass oscillate 126 times in three minutes at this point, what would be its time of oscillation if it were brought up to a distance of 6 cm.?

27. A short bar magnet is placed, at Gibraltar, perpendicular to the magnetic meridian, and "end on" towards a compass needle from which it is distant 100 cm. When the experiment is repeated at Portsmouth the magnet has to be placed at a distance of 110 cm. from the compass to produce the same deflexion of the needle. Compare the horizontal forces of the earth's magnetism at Gibraltar and Portsmouth.

28. In an experiment to find $\mathbf{M}$ and $\mathbf{H}$ after the method of Art. 172, a magnet of circular section (diameter = 0.8 cm.), mass 24.46 gm., length 6 cm., placed "end on" to a compass, produced a deflection of $10^{\circ} 58'$ when the centres were 23 cm. apart. When freely suspended the period of complete oscillation was found to be 9.04 seconds. Find the moment of the magnet and the strength of the field.

$$[\mathbf{K} = m \left(\frac{l^2}{12} + \frac{r^2}{4} \right).]$$

29. A hollow magnetised steel tube oscillates in the earth's field. ($H = 16$). Its period is 18 seconds; but when a rod of brass, the moment of inertia of which is 7000 gm. cm.², is inserted symmetrically in the hollow, the period becomes 40 seconds. Find the magnetic moment and the moment of inertia of the magnet.

CHAPTER XIX

PERMEABILITY AND HYSTERESIS

174. The magnetising effect of solenoid. A piece of iron placed in the centre of a helix of wire along which a current flows becomes magnetised. We are going to find out how the magnetisation depends on the strength of the current; whether the one is or is not proportional to the other.

Take a glass tube—about 30 cm. long and $\frac{1}{2}$ cm. diameter—and coil round it evenly a helix of insulated copper wire.

Fix this with its axis E. and W. in a horizontal position “end on” to a magnetometer (M). The helix is to be joined up to a battery through an ammeter (G) (or galvanometer), and a rheostat (R); the object of the latter is, of course, to enable us to vary the current as we wish.

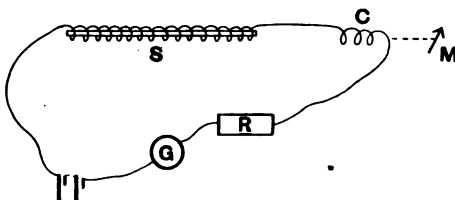


Fig. 148.

In addition to these it will be well to insert in the circuit a small compensating coil. This is shewn at C . It must be of such a size and placed in such a position that its effect on the compass needle of the magnetometer exactly neutralises the effect of the main helix (S) when any current passes through both of these in series, and there is no iron in either.

Insert in the tube a core of iron. Adjust the rheostat to give a very small current and notice the deflection (θ) produced. Increase the current gradually till the maximum is reached.

Be careful that the iron is not shaken in any way during the experiment.

Fill up a table

Current	θ	$\tan \theta$

Plot a curve to connect the first and last columns.

We know from Art. 164 that the force due to a small magnet at a point on its axis is $\frac{2M}{d^3}$ and that the deflection produced in a compass needle is given by the relation $\frac{2M}{d^3} = H \tan \theta$.

Hence $\tan \theta$ is proportional to the moment of the magnet and therefore to the intensity of its magnetisation.

The curve plotted shews the relation between the magnetic force and the intensity of magnetisation.

If a solenoid is very long in comparison with its diameter it can be shewn that (H) the force on unit pole placed *anywhere* inside it is always the same and equal to $4\pi ni$ where i is the current strength and n the number of turns per centimetre (v. Art. 139),

$$H = 4\pi ni.$$

Now if the helix surrounds a bar of iron it becomes necessary to imagine that some sort of hole is cut in the iron if we are to place a unit pole there; suppose the iron cut straight across and the pole inserted in the narrow gap so caused. Each piece is a magnet and it may be proved that if the intensity of magnetisation is I , then the magnetic force exerted at a point in the gap by the two together is $4\pi I$. (Cf. Art. 221.)

The total force on unit pole is therefore $H + 4\pi I$, where H is the force that would be exerted were no iron present.

This force is called the *magnetic induction*.

w.

12

The ratio of the intensity of magnetisation ($\mathbf{I}$) to the magnetic force ($\mathbf{H}$) is called the *magnetic susceptibility* and is usually denoted by k ,

$$\mathbf{I} = k\mathbf{H}.$$

As is easily seen from the curve plotted in fig. 149, $\mathbf{I}$ is not proportional to $\mathbf{H}$, i.e., k is not a constant.

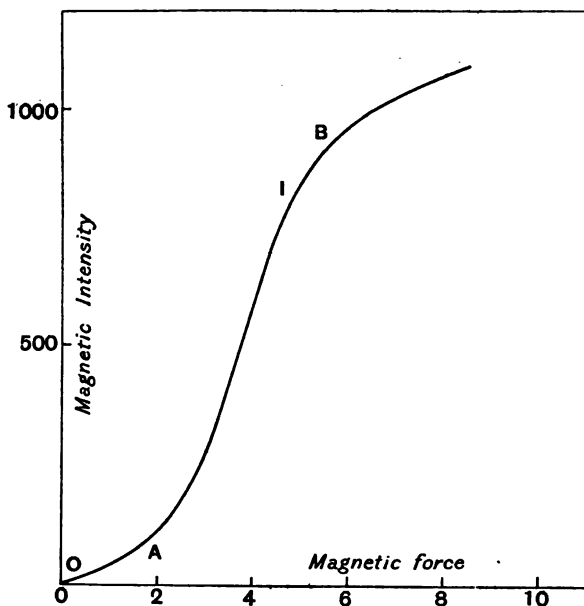


Fig. 149.

The ratio of the magnetic induction, usually denoted by $\mathbf{B}$, to the magnetic force is called the *permeability*, μ ; hence

$$\mu = \frac{\mathbf{B}}{\mathbf{H}} = \frac{\mathbf{H} + 4\pi\mathbf{I}}{\mathbf{H}} = \frac{\mathbf{H} + 4\pi k\mathbf{H}}{\mathbf{H}} = 1 + 4\pi k.$$

It is only in iron that the magnetic susceptibility is great, in nickel and cobalt it is appreciable; in most other substances it is negligible. Hence the permeability of all materials except iron, nickel and cobalt is very nearly one.

175. Hysteresis. Use exactly the same core of iron and the same apparatus as in Art. 174. Be very careful to see that the iron suffers no shock throughout the experiment.

When the current in the solenoid has reached its maximum begin to decrease it gradually by increasing the resistance in

the rheostat. Note the values of the current and the corresponding values of the deflection (θ). When the current has fallen very low break the circuit, read the deflection and then reverse the battery connections. Let the current in the new direction be increased at first very gradually indeed and afterwards rather more rapidly until the maximum current is again reached. Then, as before, decrease this to the zero value and increase it again after a second reversal (i.e. to the original direction) till it reaches the maximum again. The operation has now been carried through a complete cycle. Enter results in columns as before.

The diagram shewn in fig. 150 is plotted from readings taken in this way, though the first rise, shewn in fig. 149, is omitted. Note the following points:

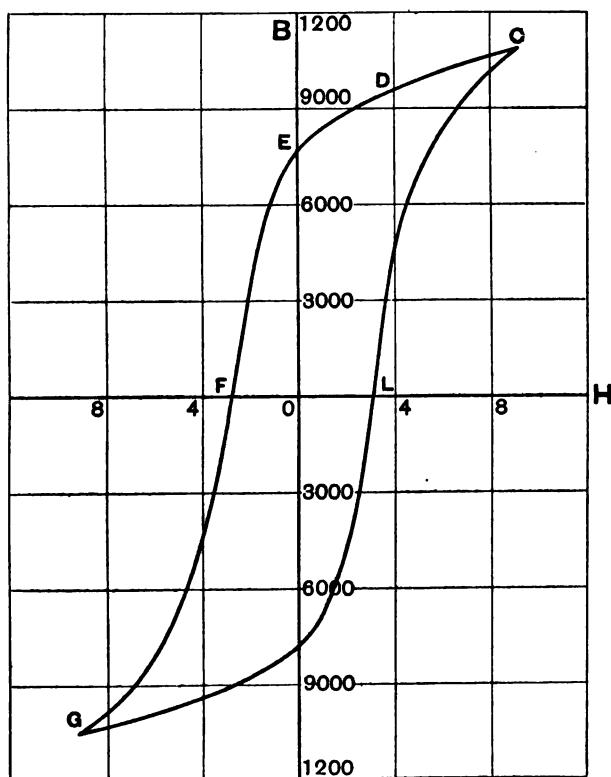


Fig. 150.

(1) when the current is very small and the iron not previously magnetised the magnetisation (which is proportional to $\tan \theta$) increases nearly at the same rate as the current. This is shewn by the part OA of the curve, fig. 149.

(2) as the current increases, the magnetisation begins to increase very rapidly (A to B) and then

(3) ceases to increase and becomes constant at C . In this state the iron is "saturated" (fig. 150).

(4) as the current decreases the magnetisation decreases also, but its value is always greater than that corresponding to the increasing current.

(5) when the current is zero the iron is still magnetised (E).

(6) when the current reverses, the iron at first is still magnetised in the old direction (E to F), but the intensity falls rapidly, reverses at F and gradually reaches a value near the maximum at G . The complete cycle gives rise to a closed curve $CDEFG$. The "softer" the iron the less will be the area of this curve and the nearer the points F and L will be to 0. It can be shewn (Art. 176) that the area of this curve nearly represents the work done on the iron in carrying it through the magnetic cycle. It is very important that for the iron used in the armature of an electric motor or dynamo this work should be small. Cf. Art. 134.

The "lagging behind" of the magnetism relative to this magnetising force is called *hysteresis*.

176. Work done in a magnetic cycle. Suppose a bar AA' of soft iron is placed in a uniform magnetic field of strength $\mathbf{G}$: that originally it is at right angles to the lines of force but is turned about an axis O at right angles both to it and to these lines till it has swept out a complete circle. At any time let AA' make an angle θ with the initial position CC' . The component of the field in the direction $OA = \mathbf{G} \sin \theta$. This is the magnetising force, usually denoted by $\mathbf{H}$. If k is the susceptibility, v the volume of the iron, $\mathbf{I}$ the intensity of magnetisation and $\mathbf{M}$ the magnetic moment in this position, then

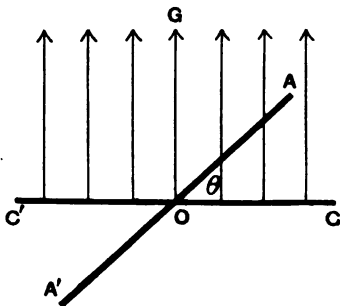


Fig. 151.

$$\mathbf{I} = k\mathbf{H} = k\mathbf{G} \sin \theta, \text{ and } \mathbf{M} = \mathbf{I}v = vk\mathbf{G} \sin \theta.$$

The couple acting on the magnetised iron due to the field $\mathbf{G}$ is equal to $\mathbf{MG} \cos \theta$ (Art. 185). Hence the work done in a rotation through a further angle $d\theta$ is $\mathbf{MG} \cos \theta d\theta$.

Hence the total work done in any finite displacement

$$= \int \mathbf{MG} \cos \theta d\theta = \int v k \mathbf{G} \sin \theta \mathbf{G} \cos \theta d\theta = \int v k \mathbf{H} d\mathbf{H} \\ = v \int \mathbf{I} d\mathbf{H};$$

the integration being taken between the limits assigned.

Now for a complete cycle $\int \mathbf{I} d\mathbf{H}$ is represented by the area of a closed curve (cf. fig. 150). This area therefore measures the work per unit volume in taking the iron through the cycle indicated.

If as is usually the case a $\mathbf{B}$, $\mathbf{H}$ curve is plotted, the area enclosed represents $\int \mathbf{B} d\mathbf{H}$.

$$\text{But } \int \mathbf{B} d\mathbf{H} = \int \mathbf{H} (1 + 4\pi k) d\mathbf{H} = \int \mathbf{H} d\mathbf{H} + 4\pi \int k \mathbf{H} d\mathbf{H}.$$

The first of the integrals is zero, if the cycle is complete, for the initial and final values of $\mathbf{H}$ are the same.

Hence the $\mathbf{B}$, $\mathbf{H}$ curve area represents 4π times the work done in taking the unit volume of the iron through a complete circuit.

EXAMPLES

1. From the following readings of $\mathbf{B}$ and $\mathbf{H}$ —both in gauss—plot a $\mathbf{B}$, $\mathbf{H}$ curve. Calculate also the permeability for the different values and plot a $\mathbf{B}$ (abscissa), μ (ordinate) curve:

$\mathbf{H}$	1	2	4	6	12	30	40	60	120
$\mathbf{B} \div 10^2$	14	48	74	96	120	150	155	164	175

(With a field of $\mathbf{H} = 24,500$ Ewing found $\mathbf{B} = 45,300$.)

2. A magnet 20 cm. $\times$ 2 cm. $\times$ 6 cm. and intensity 23 electromagnetic units lies on a table pointing N. and S. in a place where the horizontal strength of the earth's field is 31 gauss. What work would have to be done to turn it E. and W.?

3. A soft iron ring has a mean radius of 10 cm., the iron being 1 cm. in diameter. Calculate the number of ampere turns required to give a magnetic flux through it equal to 4000 centimetre-gram-second units, the permeability being taken as 2000 in the same units.

4. A solenoid of 20 turns per cm., diameter 3 cm. and length 120 cm., carries a current of 2 ampere: what is (1) the flux, (2) the strength of field inside?

5. Find the intensity of magnetisation of a core of iron placed in the solenoid of the previous question; the iron being of the kind described in fig. 149.

6. Fig. 150 is a $\mathbf{B}$, $\mathbf{H}$ curve plotted to scale; both units are measured in gauss. Find by measurement of the area enclosed the work done per unit volume in taking the iron through the complete cycle.

CHAPTER XX

ELEMENTARY STATIC ELECTRICITY

177. Electrification by Friction. Everybody knows that an ebonite pen or stick of sealing wax rubbed on the sleeve has the power of picking up light objects. Such a body is said to be electrified. To electrify a glass rod dry it before the fire and rub on a dry warm silk handkerchief. Ebonite, *dry* paper, sealing wax, sulphur are easily electrified by rubbing with flannel.

By suspending rods of ebonite or glass by means of a paper stirrup threaded by a silk fibre, verify

- (1) two ebonite rods—rubbed as above—repel one another.
- (2) two glass rods repel.
- (3) an ebonite rod and glass rod attract each other.
- (4) either rod attracts rods of wood, brass, etc. but has little effect on an unelectrified rod of dry glass, or ebonite, or sealing wax.

These experiments lead us to believe that there are two kinds of electrification: that the like kinds repel one another, the unlike attract one another and that either attracts to some extent bodies previously uncharged. The two kinds are named positive or vitreous (e.g. glass rubbed on silk) and negative or resinous (e.g. ebonite rubbed on flannel).

178. The Electroscope is a useful instrument for detecting small charges of electricity. In fig. 152, *G, G* are gold leaves which are hung from the brass rod *AB*. *P* is a

plug of wax or ebonite which supports and insulates the rod. *D* is a metal plate. The outer case may conveniently be of glass, but should be lined with wide meshed gauze which can be connected to earth. Any charged body, held near the plate, causes the leaves to diverge.

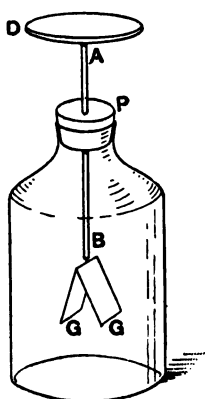


Fig. 152.

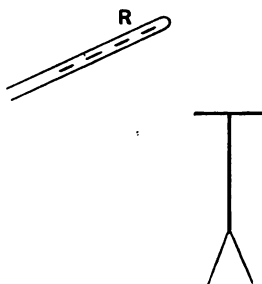


Fig. 153.

179. Electrostatic Induction. An uncharged conductor brought near to a charged body becomes itself electrified. Thus suppose a negatively charged ebonite rod *R* is held near an electroscope as in fig. 153 the latter becomes charged by induction, the near part (the plate) being positive, the far part (the leaves) being negative. It is the presence of the like (−) charges on the leaves that causes the divergence.

180. To charge an Electroscope (a) by induction. As soon as the rod in the previous article is removed the leaves collapse, and the electroscope is without charge. To charge it touch the plate with the finger while the rod is in position: this causes the leaves to collapse. Next withdraw the finger. On the final withdrawal of the rod the leaves diverge again and the electroscope is left with a charge opposite to that of the rod.

(b) *by conduction.* Charge an ebonite rod and roll it up and down on the plate. The charge acquired is of the same kind as that of the rod.

181. To test the sign of a charge. Give an electroscope a small positive charge so that the leaves diverge a few degrees. The approach of a body having a positive charge causes further divergence. This divergence is a sure test that the electroscope and the body are similarly charged. A body oppositely charged would cause the leaves to fall, but any *uncharged* body would produce the same effect. Hence if the body under test does not cause further divergence, it is well to discharge the electroscope and charge it up again in the opposite way.

182. Electrons. As yet we have no real evidence that electricity is an actual thing and not a mere state; years ago it used to be regarded not as material at all, but as a form of energy something like heat; for heat can pass from one body to another and cause rise of temperature, and yet it is not a material substance.

There is good reason to believe that electricity is not energy but is more like matter. The modern idea is that an atom of matter is built up of the two kinds of electricity, positive and negative. Negative electricity exists in the atomic form: that is it is found only in charges which cannot be divided, created or destroyed. These charges are called electrons. They are identical in all kinds of matter; their mass is about that of the thousandth of that of an atom of hydrogen.

A current of electricity involves the passage of these electrons along the conductor. In electrolysis they pass from kathode to anode in company with the ions, singly if the ion is monovalent, two together if it is divalent.

A negatively charged body is one which has more than its normal number of electrons; in a positively charged body some of the atoms have been deprived of their full complement. Thus when ebonite is rubbed on flannel there is a transfer of electrons from the flannel to the ebonite.

A conductor is a body through which electrons can pass with more or less freedom. In a non-conductor or insulator they cannot pass. Electrons repel one another: so do positive charges but electrons are attracted by positive charges. Hence bodies with like charges repel one another; with unlike they attract.

These assumptions help to "explain" electrification by induction; for consider what takes place in the charging of an electroscope. When the ebonite (—) rod is brought near to the plate, its surplus electrons which constitute its negative charge, repel the electrons in the rod and plate of the electroscope and drive some of them away to the leaves. If the hand or any other uncharged conductor is brought in contact with the electroscope, a way of escape is offered to the electrons and some of them will pass away altogether. The electroscope is thus left without its normal supply of electrons, i.e. it is positively charged. When first the hand and then the rod are removed, the electrons remaining distribute themselves afresh and the leaves diverge.

On the other hand, if the original charge had been positive, it would have pulled over electrons from the hand into the plate, and the electroscope would have acquired a negative charge.

It is always the electrons which move: the positive charge does not separate from the atom.

183. Charges produced by friction are equal and opposite. Of course if the theory is sound, this result must follow. To test it, take a rod of ebonite, roll one end of it up in a piece of silk or flannel, and rub the rod round and round. Bring both near to an electroscope. The leaves will not be affected. Now drop the flannel off the rod into a metal case placed on the plate of an electroscope; the leaves diverge. Place the rod in as well; the leaves collapse again.

184. Proof-plane. It is not always convenient to bring a body up to an electroscope when we wish to find out how it is charged. We can avoid the difficulty by using a small disc of metal held in an insulating handle. The disc after contact with the charged body is removed, but carries with it a small portion of the charge. This can be tested by an electroscope.

185. Distribution of Charge. The charge on a conductor is not distributed throughout it: it can exist only on the surface. If the conductor is hollow and empty, no charge can exist on the interior surface. Charge an insulated metal

vessel. Test the inside and outside surfaces by means of a proof-plane.

Faraday used an insulated butterfly net made of conducting gauze. He charged this but on turning it inside out he found that the charge was always on the outer surface.

186. Cavendish Experiment. Perhaps the best experimental demonstration that a charge always can only remain on the outside of a conductor is due to Henry Cavendish (1773). His apparatus, the original sketch of which is given in fig. 154, consisted of :

- (1) A globe G supported on a glass rod Ss .
- (2) Two hemispherical shells h, H which would fit together to make a sphere capable of surrounding the globe G . Notches were cut in these shells so that the support Ss of the inner could pass through.

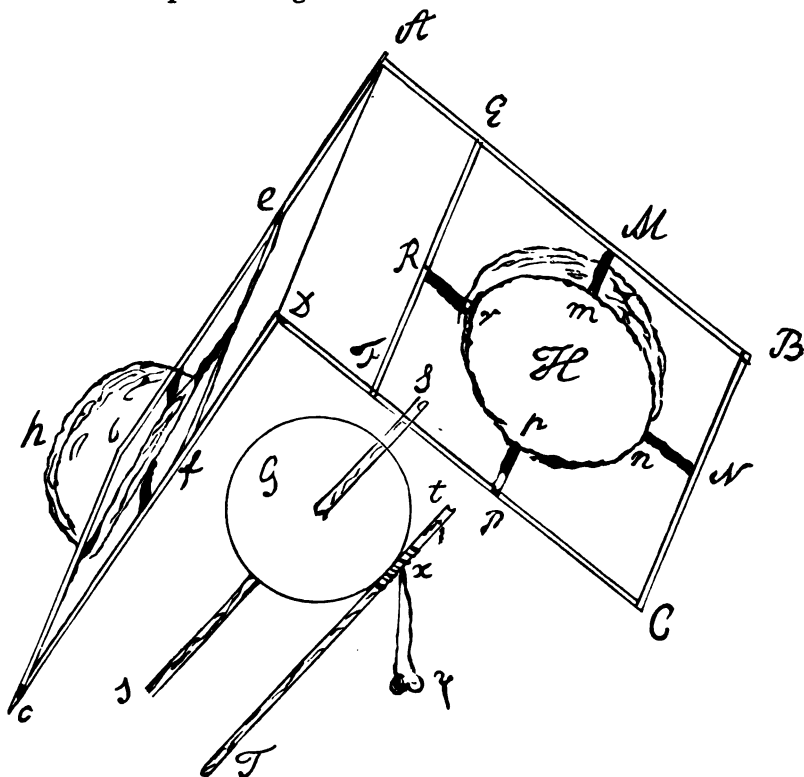


Fig. 154.

(3) A wire pushed through the shell and connecting the inside to the outside. This was threaded with silk so that it could be withdrawn if desired.

The hemispherical shells, made of pasteboard lined with foil, were supported on frameworks of wood to which they were attached by rods of glass *Rr*, *Mm*, *Nn*, *Pp*. By an arrangement of strings and pulleys the hemispheres could be lowered down and made to close over the globe; or they could be withdrawn from it altogether.

Cavendish after having electrified the hemispheres broke the connection between them and the interior globe by pulling out the wire. Then he instantly separated the two hemispheres, withdrew and discharged them. Finally he brought up an electroscope and tested the globe but found no signs of electrification. Even if the globe had been charged originally no charge could have remained on it after being surrounded by the two hemispheres and connected to them. The electroscope used was a pair of pith balls *y* hung by linen threads to a glass rod *Tt* on which was wrapped a piece of foil at *x*. A very important deduction from this result is given in Article 214.‡

187. Ice Pail Experiment. Place a light metal vessel on the plate of an electroscope. Charge a metal ball suspended by a thread of silk and bring it up to and into the vessel, noticing the behaviour of the leaves as you do so. The nearer the ball gets the more the leaves diverge, but when the ball is once inside the vessel, the leaves are quite unaffected by any changes in its position. If the ball is withdrawn without having touched the vessel, the leaves will fall. On the other hand, if the ball touches the vessel, (1) the leaves are quite unaffected by the act of contact, (2) they remain divergent when the ball is withdrawn, (3) the ball is completely discharged. Now immediately before contact, there are three charges to be considered; *A*, the charge on the ball which we will suppose negative; this will produce a positive charge, *B*, on the inside, a negative charge, *C*, on the outside. The charge *B* must be equal to *C*; they are the equal and opposite charges produced by induction. But by contact between ball and vessel, the ball

loses all its charge without producing any disturbance or redistribution of the charge on the outside, for the electroscope is unaffected by the discharge of the ball; further the inside of the vessel loses all its charge. It follows that A must neutralise B exactly, C remaining unaltered: but B is equal to C so that all three charges must be equal.

In other words, when the ball is brought up into the vessel carrying free electrons with it, it causes a displacement of the electrons in the material of the vessel; producing an extra supply on the outside and a deficit on the inside. The extra supply on the outside is exactly equal to the free electrons on the ball: therefore the ball simply carries as many electrons to the interior of the vessel as its introduction causes to pass to the outside.

188. Density. Take a conductor shaped like a pear. Insulate and charge it. By means of the proof-plane test the electrification of different parts. Bring the proof-plane into contact with the blunt end and then put it inside a metal vessel on top of an electroscope; notice carefully the divergence of the leaves. Now repeat, but touch the sharp end of the conductor. The greater divergence of the leaves indicates that the charge is more concentrated at the sharp end than at the blunt; in other words, that the *surface density* of the electricity is greater there. Do another experiment which may seem to contradict the last. Use the same apparatus but join the plate of the electroscope by a fine flexible wire to the proof-plane. Bring the plane into contact with the conductor and move it over the surface from place to place. The leaves will diverge, but the divergence will be the same whatever be the part in contact with the plane. Of course this result should be expected, for if any one point is metallically connected with the electroscope all other points are.

189. Action of points. On a charged conductor the surface density is greatest at the sharpest points. From any isolated charged body, electricity will escape in time and the rate at which it escapes is partly dependent on the surface density. Sharp points therefore accelerate the discharge. There are several ways of illustrating this. If for instance a Wimshurst or other electrical machine is worked in the dark

a luminous discharge may be seen taking place from specks of dust or from the ends of wires connected to the knobs, and for this reason a machine must be kept clean if it is to work efficiently. In an extreme case the discharge may carry air with it sufficient to blow out a candle: or to make a mill (fig. 155) revolve.

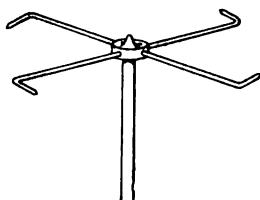


Fig. 155.

Charge a large brass ball. Now hold in your hand a pin pointing to the ball and quite close to it. Afterwards test the ball to see whether it has lost its charge.

The action here is due to the charge induced on the pin point. The charge—opposite in sign to that on the ball—passes over from the pin and discharges the ball.

On the other hand an uncharged conductor fitted with a spike will acquire a charge if it is brought close up to a charged conductor. This explains the function of the collecting combs on a Wimshurst.

190. Electrical machines. There are many forms of electrical machines. The oldest type is the frictional machine in which a revolving plate or cylinder of glass is electrified by friction against a fixed rubber. The charge so produced was collected by a comb and led to a prime conductor. Such machines are not used now and will not be described. Among newer forms the Wimshurst is perhaps the most important and useful. Volta's electrophorus, invented about 1775, is of interest and illustrates the principle of all influence machines.

191. Volta's Electrophorus consists of three parts. (a) the cake which is usually a flat disc of ebonite; (b) the sole: this is a conducting plate on which the cake is fixed; (c) the plate: a flat conducting disc to which is fixed an insulating handle.

To obtain charges, place the cake and plate on a table and electrify the cake by striking it with a catskin or cloth:

- (a) then place the plate on the cake,
- (b) connect the plate to earth by touching with the finger,
- (c) remove the plate by the insulating handle,
- (d) discharge the plate.

Repeat *a*, *b*, *c*, *d* in order.

After the cake has been rubbed it becomes electrified: we will assume negatively. The negative charge on the upper surface induces a positive charge on the upper side of the sole: the corresponding negative passes to earth. When the plate is placed on the cake, it will be charged by induction, not by conduction for the cake is a non-conductor and the plate can only make real contact with it at two or three small spots where the surfaces are uneven. The lower side of the plate will be positive, the upper negative. By contact with the finger, the negative charge escapes leaving the plate with a positive charge, nearly equal in amount to the original negative on the cake.

When the plate is removed, the positive spreads all over it and a spark can be obtained if the finger is brought near the edge.

By none of these changes has the original negative charge in the cake been affected: the operations may therefore be repeated as often as desired until the charge gradually leaks away.

Fig. 156 shews the distribution of charge after operations *a*, *b*, *c*. The presence of the sole serves to reduce the

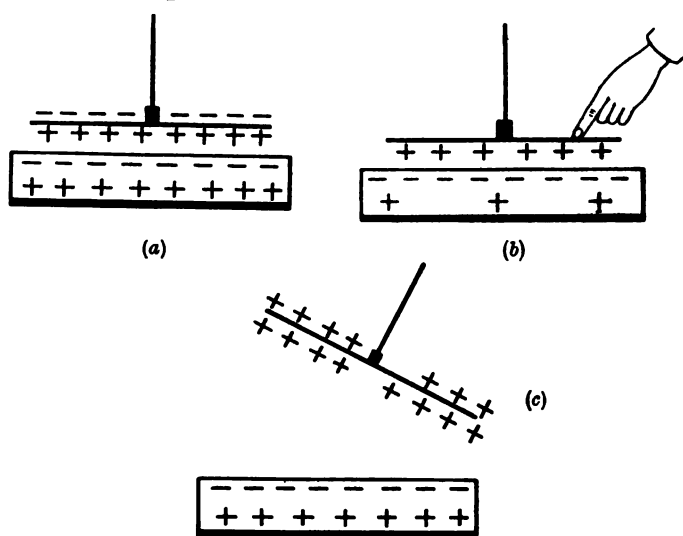


Fig. 156.

potential at points on the upper surface of the cake (*v.* Art. 202) and so tends to prevent the escape of the charge from it.

192. The Wimshurst. In this machine two circular glass plates are mounted and driven round in opposite directions on a fixed horizontal axle (fig. 157). Each plate carries a few metal sectors, *s, s, s*. *nn'* is a neutralising rod fastened to the axle; it is of metal and carries a wire brush on each end; the brushes touch the faces of the revolving sectors. There is a second neutralising rod on the other side, the brushes of which touch the sectors of the other plate. Its position is shewn by the dotted line. The collecting combs are shewn at *CC'*, fig. 158. Their spikes nearly touch the revolving sectors.

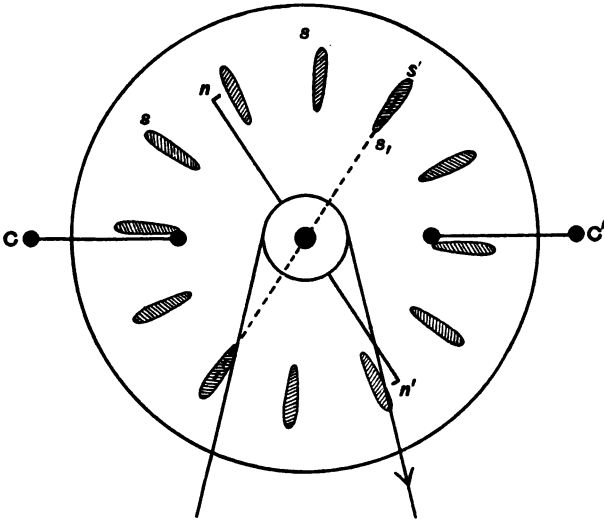


Fig. 157.

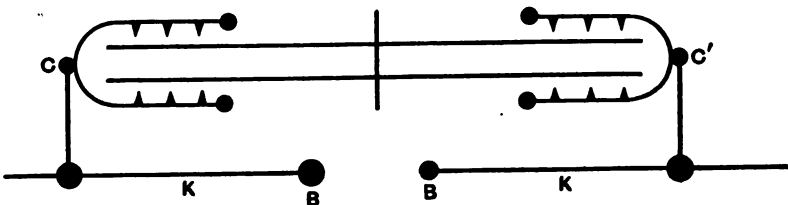


Fig. 158.

Consider the action of the front plate only. Suppose that behind this plate and close to the sector which the brush at n is touching is placed a positive charge. This will induce charges on the rod and sector, negative on the sector, positive on the rod. As the plates revolve the sector leaves this brush and passes on: the charge it carries cannot escape till it comes to the comb C' . Each sector will behave in the same way, giving C' a succession of negative charges, so that C' will become negatively charged.

Now the sector s' , having come from n , is charged negatively. It will influence the sector s_1 (not shewn in figure) opposite to it on the other plate; in the position shewn, s_1 is in contact with a neutralising brush, and therefore acquires a (+) charge. As the plate rotates it carries this charge behind n in the direction nC and delivers it to the collecting comb C . Hence a succession of (+) charges passes to C from the back plate; just as (-) charges pass to C' from the front plate.

As soon as the machine is started, the presence of the initial (+) charge behind n becomes unnecessary, for the (+) charges on the sectors of the back plate will supply its place.

We have only described the action of the upper halves but the lower behave in exactly the same way.

The collecting combs, mounted on insulating supports, are joined to adjustable discharging rods KK and balls BB .

Usually the rods are connected to the inside coatings of a pair of Leyden jars (Art. 201), the outsides of which can be joined together. When this is done the sparks are much longer and sharper, though less frequent.

193. Easy practical exercises.

1. Test the sign of dry paper rubbed on the sleeve, the palm of hand, silk, sheet rubber.
2. Electrify a brass rod held in an insulating handle and test the sign of the charge.
3. Hang a pith ball on a fibre of silk. Hold on one side of it a (-) ebonite rod, on the other a (+) glass rod. Observe and account for the motions.
4. Hang two hollow brass balls in contact with one another by threads of silk. Bring a charged rod near. Then

separate the balls and remove the rod. Test the electrification of each. Then let the balls touch and again test.

5. Hold a charged rod above a pith ball lying on the table. Notice the behaviour of the ball. In what respects is the behaviour altered if the ball is laid on a slab of wax instead of on the table?

6. *Conductors and Insulators.* To find if a material is a conductor or an insulator charge an electroscope, hold the material in the hand and touch the electroscope with it. A good conductor will discharge it instantly; with a good insulator there is no visible change; with a poor insulator the leaves will fall gradually. Among good insulators are wax, dry glass, ebonite, resin, silk, china. Test these and also chalk, wood, cotton, wool, carbon, paper in its ordinary condition, well dried paper. Devise a method to test liquids and use it for water, paraffin. Gases under ordinary conditions are insulators: try however the effect of bringing a lighted match near the plate of a charged electroscope.

EXAMPLES

1. Two unequal brass balls, suspended by silk threads, are charged. By means of an electroscope only, how would you find out whether the charges were alike or not? Could you find out if they were equal or not?

2. A pad of flannel is placed at the bottom of a metal vessel, which is insulated and connected by a wire with the cap of a gold leaf electroscope. One end of a long rod of sealing-wax is now rubbed against the flannel. What indications will the electroscope give (1) while the rubbing is going on, (2) when the sealing-wax is withdrawn?

3. A charged electrophorus is capable of supplying an indefinitely large amount of electricity; explain why the charge it can convey to an insulated sphere is practically limited.

4. A charged electrophorus has energy. What is the source of this energy?

5. Can the leaves of an electroscope diverge when the knob is connected to earth, and if so how could the divergence be prevented?

6. A thimble is charged with electricity. How will the charge be distributed?

7. When two different substances are rubbed together electric charges are produced. Explain why the presence of these charges is not always apparent.

8. *A* and *B* are two neighbouring spheres suspended by silk threads. *A* is charged with + electricity, and while it is charged *B* is touched with the finger. *B* is then left insulated and *A* is touched; what is the electric state of *A* when the finger is removed from it?

9. Two conductors *A* and *B* are insulated and placed near each other. *A* is strongly charged with positive electricity; *B* is uncharged and has a point projecting from it. How will the charge of *A* be affected by the presence of *B*; when the point is turned (1) towards *A*, (2) from *A*?

10. If two equal raindrops, charged with equal quantities of electricity of the same kind, were united into one large drop without losing any of their charge, what would be the ratio of the surface densities of the charges before and after union?

11. In what circumstances is it possible to transfer the whole of the charge on a conductor to another insulated conductor?

12. Two insulated metal vessels are placed one inside the other. (*a*) A positively charged brass ball is introduced into the inner, but not into contact; (*b*) the outer vessel is momentarily touched; (*c*) the ball touches the inner vessel; (*d*) the inner vessel is touched. Describe the distribution, and changes in potential after the operations *a*, *b*, *c*, *d*.

13. A tin can is placed on a block of wax. Another can is placed inside this and is insulated from it. (*a*) A positively charged brass ball is introduced into the inside without contact, (*b*) the two cans are then connected, (*c*) then disconnected, (*d*) the ball is removed, (*e*) the inner can is taken away from the outer.

Shew, with sketches, how the various bodies will be charged after each of the operations *a*, *b*, *c*, *d*, *e*.

14. At what times in using an electrophorus is work done (*a*) *by* the operator, (*b*) *on* the operator?

CHAPTER XXI

POTENTIAL AND CAPACITY

194. Force diagram. By the electric force or intensity at a point is meant the force that would be acting on a unit positive charge placed at that point. Take the case of a charged sphere. At a great distance the force due to it on a unit positive charge would be very small: as the distance decreases, the force increases, being four times as much at half the distance. Fig. 159 is a diagram in which the height of the ordinate represents the force at a distance measured by the abscissa.

If the sphere is a conductor there can be no force inside: hence the gap *ab* in the curve.

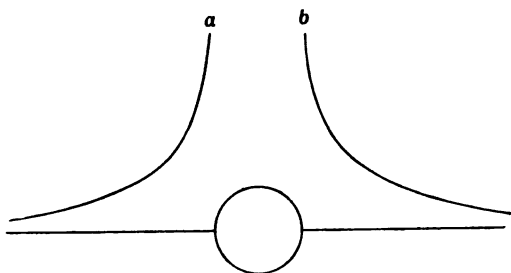


Fig. 159.

195. Potential. Difference of potential has been fully dealt with in Current Electricity, but the actual potential at any point was seldom required. It was, however, referred to in Art. 42.

The potential at any point—either in an insulator or in a conductor—is measured by the work necessary to bring a unit positive charge of electricity to that point from a place where the potential is zero: i.e. from a very great distance or from the earth. In Static Electricity we speak usually of “potential *at* a point” or “the potential *of* a conductor.” We do this because the potential at all points on a conductor is the same—for if the points on a conductor were not at the same potential, a current would flow from one to the other as we already know by Ohm’s Law. But an insulator will not permit a current to flow so that the potential may vary from point to point.

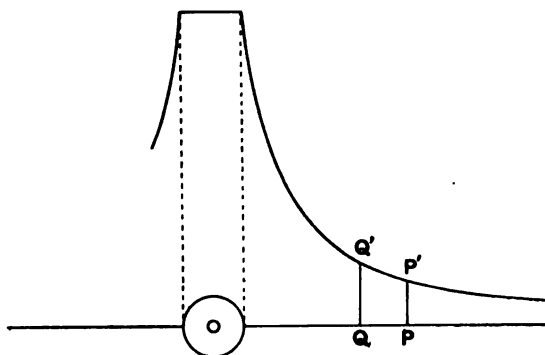


Fig. 160.

Take a very simple case. Suppose a sphere holds a positive charge. At a great distance from the sphere, the force due to its charge is small and therefore the work done in pushing a unit positive charge one centimetre nearer to the sphere is small: the potential therefore changes very slowly from point to point. On the other hand at places close to the sphere the force is great and the potential changes quickly. The diagram (fig. 160) illustrates this. The charge is supposed to be on the sphere *O*. The ordinates represent the potentials of the points at their feet. Thus the potential at *P* is represented by the ordinate *PP'*; and the difference in potential between the places *P* and *Q* is represented by the difference between the ordinates *PP'*, *QQ'*.

196. Potential due to equal unlike charges. Let us consider next how the potential varies from point to point along a straight line joining the centres of two spheres, one carrying a positive charge, the other an equal negative. At distant places the force due to one charge is almost counter-balanced by the force due to the other and therefore the work done in pushing up a charge from infinity to the point *A* (fig. 161) is very small: i.e. the potential at *A* is small. Neither does it increase rapidly till we reach points close to the sphere. The potential is constant throughout the sphere and we therefore get a horizontal part *BC* in the graph.

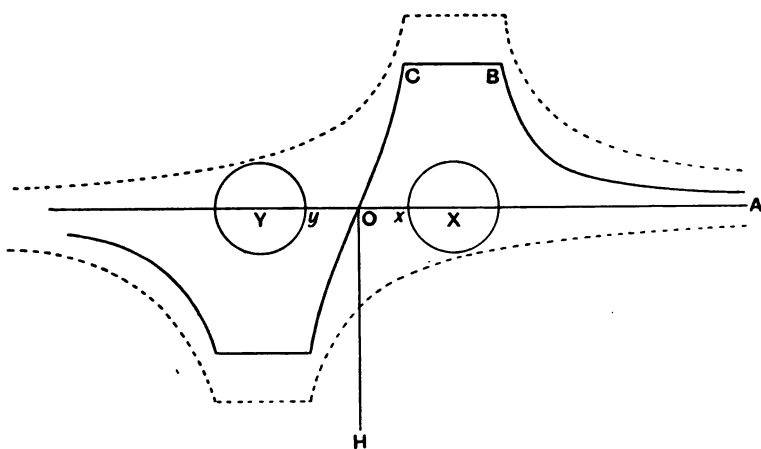


Fig. 161.

Now at all points in the line joining the centres of the spheres the direction of the force on our imaginary unit charge is always from the plus to the minus, so that no work would have to be done to push it from *x* to *y*; on the contrary the electrical forces would themselves do work; the potential therefore decreases and will fall to zero at *O*, the middle point. We can see that the potential at *O* is necessarily zero, for if the unit charge were brought up from a great distance to *O* along the line *HO* the attraction due to *Y* would always be counteracted by the repulsion due to *X* (the resultant of these equal forces being at right angles to *HO*), so that no work would be done at any point of the journey.

(It is quite legitimate for us to choose the most convenient way of approach to O : for the work done is quite independent of the particular path chosen.)

The remainder of the graph is exactly similar, but below the line of centres for the potential is always negative on the left-hand side of O .

The dotted curve above the line indicates the potential due to the presence of X and its charge: the dotted curve below the line is for Y . From the two the continuous line has been drawn.

197. Potential due to two spheres, one uncharged.

Another case is important: two spheres X and Y (fig. 162), near together, X carrying a positive charge, Y being uncharged but under the influence of X so that on the side near X , Y

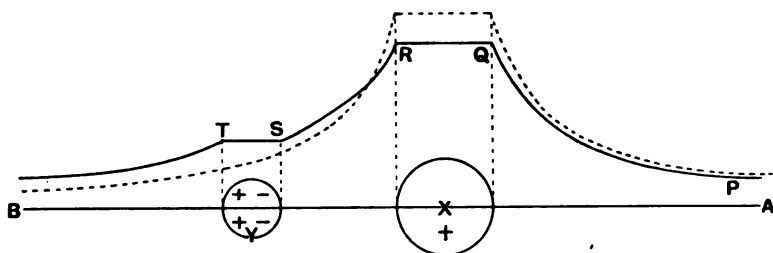


Fig. 162.

will have an induced negative charge, on the far side, an equal positive. The presence of Y will disturb the charge on X and will pull it over slightly to the side near Y .

Consider now the force that would act on a unit charge at A . It is due to three causes, (1) the plus charge on X , (2) the minus charge on Y , (3) the plus charge on Y .

Now the effect of X is rather less than it would be were Y not present, for on the whole the charge is more distant.

The negative charge on Y is rather nearer than the positive and therefore the two together will produce a slight attraction on a positive charge at A . The total repulsion on a positive charge at A is then rather less than it would be were Y absent.

As the force is less it will require less work to push a unit charge up to X . If then the dotted line shews what the

potential would be at different points were Y absent, the curve shewing the actual potential will always be below it as indicated by the line PQ . The potential is constant throughout the conducting sphere X , so that we get the horizontal part of the graph, QR .

Again at all points between X and Y the electric force is from X to Y , the potential must therefore fall continuously: this is shewn by the curve RS .

To trace the remainder of the graph, think what would happen were a unit charge brought up from the other side.

At B the force due to X is greater than it would be were Y not present; Y also on the whole exerts a small repulsion. The resultant force is therefore increased by the presence of Y : and the potential also.

We come then to the following conclusions :

- (1) The potential is everywhere positive.
- (2) The potential at all points on the side of X remote from Y is decreased by the presence of Y .
- (3) At places on the B side of Y , the potential is raised by the presence of Y .
- (4) The nearer Y is to X , the higher the potential of Y , the lower the potential of X .
- (5) If Y instead of being insulated had been connected to earth its positive charge would have left it: its potential would be zero and that of X and all places in its neighbourhood reduced.

198. Experiments on potential. An electroscope is really an instrument to indicate differences of potential: its leaves diverge when its potential differs from that of its surroundings. If then the surrounding cage is earthed, the leaves diverge if the potential of leaves and knob is not zero: if the leaves are kept at zero potential, they can only diverge if the cage is brought to some other potential. The greater the potential difference between leaves and cage, the greater the divergence: but of course the one is not proportional to the other.

Do the following experiments.

- (1) Bring an uncharged insulated brass ball near the knob of a positively charged electroscope. The leaves fall as

the ball approaches, and therefore their potential diminishes. This shews experimentally that the approach of an uncharged conductor to a charged body lowers the potential of the latter.

(2) Bring an uncharged electroscope up to a positively charged body ; or the body to the electroscope if more convenient. The increasing divergence of the leaves indicates that the potential of an uncharged body is increased by the presence of a positively charged body. These results merely illustrate (4) of Art. 197.

(3) Bring the hand—or any other uncharged uninsulated body close to the knob of a charged electroscope. The fall is greater than in (1). This verifies (5) in Art. 197.

(4) Repeat the experiment of Art. 188.

Of course as soon as the electroscope is once connected to any part of the conductor the two come to the same potential which cannot be altered by changing the position of the point of contact.

199. Capacity. The capacity of a bottle can be measured by the number of grammes of water it will hold: but if water were compressible we could only make such a measure satisfactory if we stated the pressure. The capacity of an oxygen cylinder is measured by the number of cubic feet at atmospheric pressure which can conveniently be compressed into it.

Now the amount of electricity that a body is capable of holding on its surface is not definite: we define then the capacity of a body by dividing its charge at any time by its potential. The potential of a body is however dependent on its surroundings: if then we speak of the potential of a sphere we imagine that sphere to be far removed from all other bodies. The definition given assumes that the potential of a body is proportional to its charge ; otherwise the capacity would not be constant. That this assumption is true is obvious, for of course the force exerted by a body on a unit positive charge is proportional to its charge and therefore also the work necessary to bring the unit charge from infinity to the body. The capacity of any body is dependent on shape and size.

200. Experiment. Fix a sheet of foil on a rod of ebonite after the fashion of a roller blind, and join one of the free corners by a light wire or cotton thread to a small electroscope. When the foil is open give it a charge. The leaves of the electroscope diverge. Now roll the foil up on the rod: the leaves diverge further, indicating that the potential of electroscope and foil is rising and the capacity

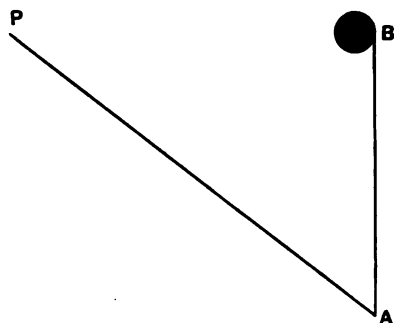


Fig. 163.

therefore falling. We expect this from the theory; for let AB represent the open foil seen in section. Consider the force that would act on a unit charge at P ; the charge on the part near A would exert a repulsion directed along the line AP , and therefore the work done in pushing up the unit charge along the line PB to the foil would be less than if the foil and its charge were gathered up to B : for then the repulsion would be both direct and greater.

201. Leyden Jar. A Dutchman, Musschenbroek, who lived in Leyden, tried to accumulate electricity in a bottle. He filled the bottle with water, corked it up and ran a wire through the cork to make contact with the water inside. Perhaps his idea was that he could lead the electricity in through the conducting wire to the water, but that the electricity could not escape again through the non-conducting glass of the bottle. A friend, who assisted him, held the bottle in one hand, placed the wire near the knob of an electrical machine and so obtained a charge. Then, with the other hand he touched the wire. He got a shock which much surprised him. Musschenbroek also had a shock and

was so badly hurt that he said he would not take another for the kingdom of France, but this was probably the effect of a lively imagination for if you repeat the experiment you will probably not get a shock big enough to be in any way unpleasant; at any rate it will do no harm.

The essentials to this experiment were two conductors (the hand round the bottle and the water), separated by an insulator (the glass of the bottle). One of these conductors, the hand, was earth connected. Any apparatus with these essentials is a Leyden jar, or, as it is often termed, a condenser. Two boys, *A* and *B*, back to back but separated by a slab of wax or a sheet of dry glass will make a condenser if one of them, *B*, stands on an insulating stool. If *B* puts his hand quite close to the knob of a Wimshurst he will receive a series of sparks. If *A* and *B* then touch hands, a spark will pass between them and they will receive a mild shock. The theory of the condenser is simple and depends on work we have already done.

202. An **Epinus condenser** consists simply of two metal plates; one *A* fixed, the other *B* movable. These can be separated by a third plate *C*, made of glass or other insulator. For the present we will leave out plate *C* so that the two conductors which form the plates of the Leyden jar are separated by air. Both plates *A*, *B* are mounted on glass standards. Connect *A* to a small electroscope *E*, *B* to the earth.

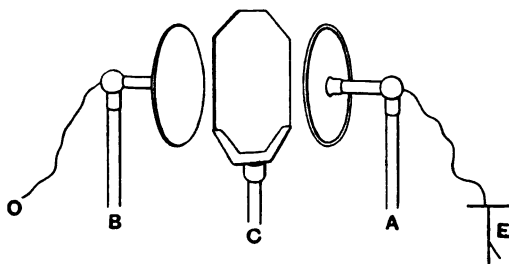


Fig. 164.

Let the two be separated as far as possible: then give *A* a charge sufficient to make the leaves of *E* diverge. Bring *B* closer to *A*. The leaves will begin to collapse, shewing that the potential of *A* and *E* is decreased by the approach

of *B*. Give *A* a further charge till the leaves of *E* diverge as much as at first. Again bring *B* closer and repeat the operation. It will be found that *A* can take a very large charge before *B* gets quite close to *A*. Yet if the divergence of the leaves of *E* remain the same as before, the potential of *A* and *E* must be the same as at first. Hence the condenser produces the effect indicated by its name: it enables us to store a large charge of electricity without a high potential. In other words it has a large capacity.

In this experiment there is of course a limit to the nearness of *A* and *B*: if they are too close a spark will pass between them. There is also a limit to the charge that can be given to *A*. The charge is always escaping somehow or other so that after a certain point it leaks as fast as it is supplied.

Another way of looking at the action of the condenser is this. Suppose *B* is insulated and brought close to *A*. The charge which *A* receives goes partly to the leaves of the electroscope and causes them to diverge. But *B* becomes charged by induction. If *A*'s charge is positive, the near side of *B* is negative, the far side positive. Now touch *B* for a moment. Its positive charge will pass away to earth: *B* will be left negative and will pull up the positive electricity on *A* and *E* nearer to itself: some of the positive will then leave the electroscope so that the leaves will collapse to some extent. An additional charge will be shared by *A* and *E* as before, the leaves diverge again and the far side of *B* will get a new positive charge by induction. Touch *B* and the cycle repeats.

203. To make a Franklin plate. Take two pieces of tinfoil, and fasten them by shellac varnish one to each side of a pane of glass. The foil should reach about an inch from each edge of the glass.

Warm the plate, and cover the exposed part of the glass with shellac.

Connect one of the coatings to earth, and charge the other either by a machine, or by repeated applications of the electrophorus. A shock can be obtained by touching both coatings at once.

204. Experiments with a Leyden jar. To charge a jar, the outside should be connected with the earth, and the knob which is usually joined by a chain to the inner coating should be placed near—but not quite in contact with—one of the knobs of a Wimshurst or other kind of electrical machine.

(1) Charge a small jar slightly: touch the knob with the finger and you will get a quite perceptible shock. With a large jar fully charged the shock is painful and dangerous.

(2) Let several boys join hands and make an unbroken line. One of the end boys holds the outer coating of a small charged jar. When the other end boy touches the knob a shock will pass along the whole line.

(3) Charge a jar fully: discharge it by means of the proper insulating discharging tongs. The spark will be large enough to light a gas.

(4) Charge a jar fully: then insulate it by placing on a slab of wax. If you now touch the coatings alternately—never both at the same time—you will get a large number of small shocks. This is called the discharge by alternate contact.

205. Discharge by alternate contact. Suppose we had two circular conducting discs exactly alike, far removed from one another. Let one of them, *A*, have a positive charge and the other, *B*, an equal negative charge. On neither would the charge be uniformly distributed; the density would be greatest at the edges, least at the centre; but the distribution would be the same on both sides.

But now suppose the plates are brought close together. The distribution will be much altered: on *A* the charge will pass mainly to the side facing *B*: only a little will be left on the other side. The potential of *A* will be much decreased by the presence of the negative charge on *B*, but it will of course remain positive. The potential of *B* will be raised by the presence of the positive on *A*, but will still remain negative and equal numerically to that of *A*. The potential of any point half-way between the plates will be zero.

Now suppose one of the plates, *B* say, is connected to earth. Its potential must rise to zero and negative electricity pass from it to the earth. The potential of *A* will rise

slightly owing to the departure of this negative charge in its neighbourhood. Now insulate *B* and earth *A*. The potential of *A* must fall to zero; some of its charge passes to earth. In *B* the potential falls from zero to something negative, for a positive charge has left its neighbourhood. Now insulate *A* and earth *B*, and so on. The charge which is taken from either plate gradually gets less and less: the potentials gradually approach zero: but the plates cannot be entirely discharged in this way by a finite number of contacts.

206. Identity of static and current electricity.

The electricity called into evidence by rubbing ebonite on flannel is exactly the same thing as the electricity driven through a wire by a dynamo. But the first is usually met with in very small quantities at high potential, the latter in much bigger quantities at low potential. Indications of the identity of the two may be found from the following considerations:

(1) Insulators in static electricity are non-conductors of current.

(2) If the voltage of current electricity is raised by means of a transformer (e.g. the induction coil) the usual effects produced by a Wimshurst can be obtained. Thus vacuum tubes can be lighted, Leyden jars charged, X-rays excited either by a Wimshurst or a coil.

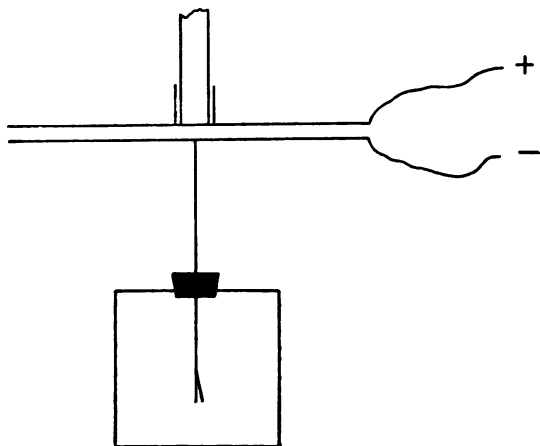


Fig. 165.

(3) Magnetic effects are not readily obtained from a Wimshurst. It has however been shewn conclusively that a static charge revolving in a circular path affects a compass needle at the centre.

(4) A static charge may be obtained from a cell. To shew this a *condensing electroscope* is often employed. It consists of an ordinary electroscope fitted with a large disc in place of the usual small plate or knob. Over this plate and insulated from it only by a film of varnish is laid a second plate. The two are connected for an instant one with the positive, the other with the negative pole of a cell. When the connections have been removed, the upper plate is raised. This causes the leaves to diverge and indicates that the electroscope has acquired a charge from the cell. The reason of the divergence of the leaves is explained fully by Art. 202.

(5) Two gold leaves, hung side by side, insulated from one another but joined to the terminals of a battery of cells attract each other.

207. Condensers. The capacity of a Leyden jar is rather small but it is possible to charge a jar up to a very high potential before the charge begins to leak rapidly through the glass: if charged to too high a potential, however, the glass may be pierced.

For many purposes a condenser of much greater capacity is required: e.g., in telegraphy and in the primary of an induction coil: but the potential difference necessary will not be nearly so great. Such a condenser can be formed of alternate layers of paper and tinfoil (*v.* fig. 88).

208. Comparison of condensers. The most direct method of comparing condensers is perhaps by means of a

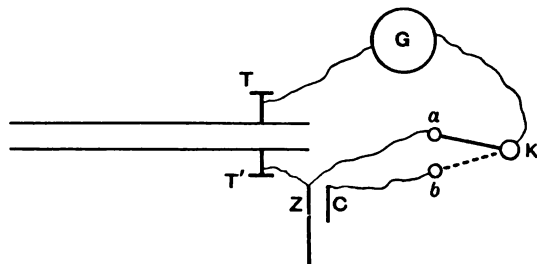


Fig. 166.

ballistic galvanometer. A diagram of the arrangement is shewn in fig. 166.

One terminal (T) of the condenser is connected to the galvanometer (G), the other T' to one of the poles (Z) of a battery. Z , and the remaining pole (C) are connected to the studs, a , b , of a two-way key or switch (fig. 167), the arm of which is joined up to the galvanometer.

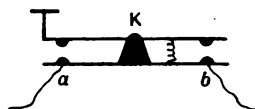


Fig. 167.

Now if K makes contact with a , both sides of the condenser must be at the same potential as Z , but when K is switched over to b , a current must flow along G to charge up the condenser until the potential difference between the plates is equal to that of the battery. To find the capacity of the condenser it is therefore necessary to know (1) the total quantity of electricity which passes through G , (2) the potential difference between the poles of the battery.

In practical work join up the apparatus as shewn using a single Daniell as a battery. Notice the position of the reflected spot of light on the scale. Then, when steady, switch the key over and so charge up the condenser. The light will move across the scale: the extreme position to which it moves must be noted. If it should happen that the "throw" is insufficient for accurate measurement, replace the single Daniell by a battery of two or more cells joined in series.

The mirror will swing for some time: when it comes to rest again discharge the condenser through the galvanometer by joining up K to a . The light will travel in the opposite direction. Note again, the extreme position. Take half the distance between these two extremes as the "throw."

Now if you wish only to *compare* two condensers, replace the first by the second and repeat the operations, without changing the battery. The capacities are nearly proportional to the "throws," for both condensers were charged up to the same potential difference (that of the battery), and the charges to do this were measured by the "throw" of the galvanometer.

If, on the other hand, you want the actual capacity you must know what charge produces a throw of one centimetre

on the scale; also, the E.M.F. of the battery. The first can be found, either from the formula of Art. 98, or by the experiment of Art. 136. The second by any of the usual methods.

The capacity measured in *farads* is then found by dividing the charge in coulombs by the E.M.F. in volts. The result will be extremely small. Multiply it by a million and the result is in *microfarads*.

EXAMPLES

1. How would you determine whether two condensers are of equal capacity or not?
2. If one of the knobs of a Wimshurst machine be placed near an insulated conductor sparks pass between them when the machine is working; after a time these cease. Explain the reasons.
3. A charged ball is suspended by a silk thread midway between two other fixed balls, the centres of all being in the same horizontal line. The fixed balls gradually receive charges. How will the suspended ball be affected?
4. How can the potential of a conductor be altered without altering the charge on it?
5. What changes in the potential of the plate of an electrophorus take place when it is used to charge a Leyden jar?
6. A charged body is held close to an iron mantelpiece. A proof plane is made to touch the part of the iron nearest the body and is then removed to a distance. What is now the state of the proof plane as regards charge and potential?
7. Describe an experiment to prove that two parts of the same conductor may be differently electrified although they are at the same potential.
8. An insulated metal plate is charged to a certain potential. Another equal and parallel insulated plate at zero potential is brought near the first, touched with the finger, and removed: what changes take place in the potential of each plate during the process?
9. A positively charged brass ball is suspended from a silk thread and brought near to an electroscope. How will the potential of (a) the electroscope, (b) the ball change?
10. An electrified body is brought into the neighbourhood of (a) an insulated conductor, (b) an earth connected conductor. Describe exactly the effect on the potentials of the electrified body and of the unelectrified conductors in each case.

11. Suggest a practical method of finding out which of two bodies has the greater capacity.

12. What is meant by saying that the earth is at zero potential?

13. Two charges are placed at A and B . Shew how the potential changes at points in the line AB , and in AB produced in both directions, in the following cases:

- (1) both charges positive and equal,
- (2) equal but opposite charges,
- (3) charge on $A = 2$ charge on B and both +,
- (4) charge on $A = 2$ charge on B , but A + and B -.

14. Explain, by theory of potential, why charge must accumulate at points.

15. Two equal insulated uncharged spheres, B and C , are placed on opposite sides of and at equal distances from a charged sphere A . What is the electrical state of B and of C , and what will happen if the part of B nearest to A is connected by a fine wire with the part of C farthest from A ?

16. An electroscope is placed within an insulated metal jar, and is insulated from it. The electroscope is connected with another electroscope at a distance. The jar is then charged with positive electricity. Describe and explain the indications of the electroscopes.

17. A metal plate is placed on the top of an electroscope. A long watch-chain, charged with electricity, is supported at one end by a fibre of silk and gradually lowered on to the plate until it coils up in a heap upon it. Explain the behaviour of the electroscope.

18. Draw a graph similar to fig. 162 for the case where Y is connected to earth.

CHAPTER XXII

LINES OF FORCE

209. Lines of force. In magnetism and current electricity lines of magnetic force may easily be traced in a strong field by means of iron filings: no such simple method is available in static electricity for tracing lines of electric force. Yet since the law of the inverse square holds between charges as it does between poles, the lines of force must often be similar. There are these differences: in magnetism we cannot separate the North Pole very far from the South: in static electricity the positive electricity may be removed so far from its complementary negative as to be practically isolated from it altogether. Again in static electricity the lines cut all conductors at right angles: they end at conductors and cannot exist in their interiors. With magnets the lines are seldom at right angles to the magnet face: they form closed curves joining North to South through the body of the magnet as well as through the surrounding air.

In electrostatics lines of force might be drawn by finding tangents at various points (Art. 146): a laborious process. A much quicker way is given in Art. 211, but even by that method the process takes much time.

Look at figs. 168, 169. Suppose that at *A* and *B* equal charges are placed: the dotted curves shew the lines of force: the continuous lines are lines of equipotential (Art. 211). Fig. 168 gives the left-hand portion of half the field when the equal charges at *A* and *B* are unlike. Fig. 169 gives the right-hand portion when they are alike. Compare these figures with the lines between like and unlike magnetic poles.

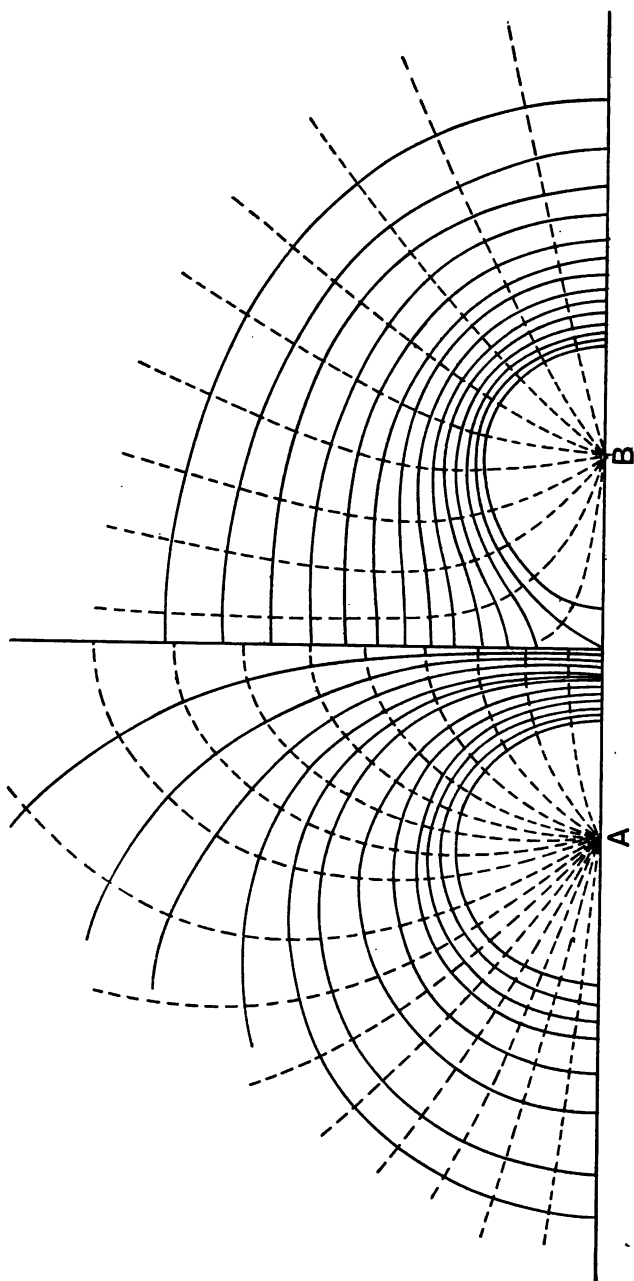


Fig. 169.

Fig. 168.

In these and all other figures note that the lines start out at right angles to the conductors.

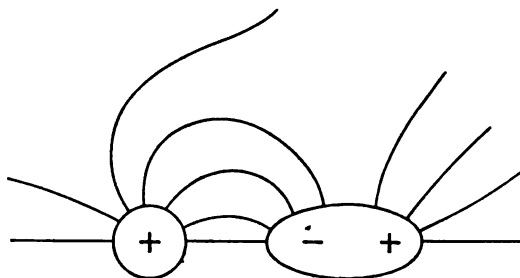


Fig. 170.

Fig. 170. Here we have a positively charged ball placed near an uncharged conductor. Note that as many lines are drawn to leave the conductor as enter it.

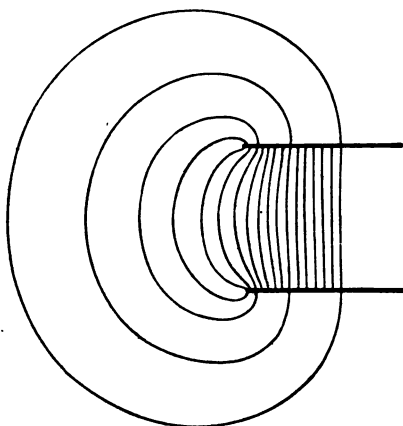


Fig. 171.

Fig. 171 shews the lines near the edges of the plates of a condenser.

210. Tubes of force. Now in Chapter XIII we said it seemed as if the magnetic lines which linked two conductors acted like elastic bands drawing them together: but that the lines also repelled one another. If we imagine the electric line of force to have the same properties, we can account for the repulsion or attraction between charges under the inverse square law. We no longer have action at a distance: it is

action throughout a continuous medium. Consider how this hypothesis would account for the attraction and repulsion in the cases represented in figures 168—171.

It is to Michael Faraday that we owe this conception. He regarded the space between charged bodies as filled with "tubes of force." These tubes were in tension along their length and under pressure at right angles to their length: that is to say they tended always to contract and to push one another apart. Only in an insulator can they exist: in a conductor they collapse at once.

To account for all the known laws of electricity we must define a tube of force more exactly. On any charged conductor take an area of such size that the charge on it is one positive unit: from points on the boundary of this area draw lines of force. Eventually these lines will end up on a second surface and enclose there an area on which is unit negative charge. The space then which is bounded along its length by lines of force and on its ends by unit charges, one positive, the other negative, is a unit tube.

The number of tubes therefore that leave any body measures the number of units of positive charge on that body: the number of tubes that end measures negative charge. In the case of any closed surface the number of tubes leaving it exceeds the number entering it by the number of units of charge enclosed.

211. Equipotential surfaces, as the name implies, are surfaces at all points of which the potential is the same. Any conductor has the same potential at all points; hence the surface of a conductor is an equipotential surface.

Now if the potential at *A* is the same as at a neighbouring point *B*, it follows that the electrical intensity in the direction *AB* is zero; otherwise work would be done against it in taking unit charge from *A* to *B*. The electrical intensity must therefore be at right angles to *AB*. Hence the electric intensity is everywhere at right angles to any line drawn through *A* to a neighbouring point at the same potential: in other words lines of force cut equipotential surfaces at right angles.

Now it is sometimes easy to draw equipotential surfaces:

in such cases curves normal to those are lines of force. By way of illustration take the cases of charges of 2 and 1 placed at points A and B , 10 cm. apart. The potential at any point due to a charge Q at a distance d is Q/d (Art. 219). Hence if a point C is 8 cm. from A and 12 from B , the potential at C must be $\frac{2}{8} + \frac{1}{12}$; and the potential at any other point may be found in the same way. To get a series of points all of the same potential, describe two sets of circles. In the figure illustrated, the first set of circles of which A is centre has

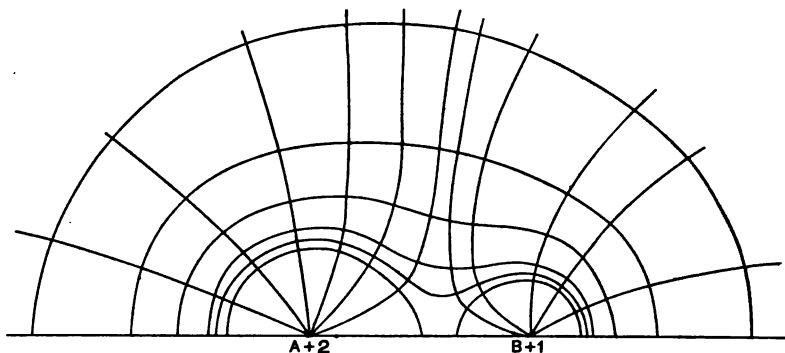


Fig. 172.

radii $2, \frac{2}{9}, \frac{2}{8}, \dots, \frac{2}{1}$ cm., so that the potentials due to the charge at A at points on these circles are 1, $\cdot 9$, $\cdot 8$, ... $\cdot 1$. The second set, centre B , has radii $1, \frac{1}{9}, \frac{1}{8}, \dots, \frac{1}{1}$, so that the potentials due to the charge at B at points on these circles are also 1, $\cdot 9$, $\cdot 8$, ... $\cdot 1$.

If now we wish to trace the locus of points the potential of which is let us say $\cdot 5$, we find, (if they exist), the points of intersection of the circles marked $\cdot 2$ and $\cdot 3$; $\cdot 3$ and $\cdot 2$, etc.

The other points marked on this particular curve are obtained by finding the points of intersection of the circles of potential $\cdot 35$ and $\cdot 15$; $\cdot 25$ and $\cdot 25$, etc.

212. Properties of lines of force. The following properties should be remembered:

(1) They start from places positively electrified and end on places negatively electrified.

- (2) Two lines of force cannot cross.
- (3) They cannot exist in the interior of a conductor which contains no charged body.
- (4) Every line of force is a line of decreasing potential.
- (5) A line cannot begin and end on the same conductor.
- (6) They meet surfaces of equipotential (and therefore all conductors) at right angles.
- (7) The lines inside a conductor are quite independent of those outside, and vice versa.

213. Measurement of charge. The electroscope affords a rough means of comparing charges. Suppose a metal vessel is placed on the cap of an electroscope, and two equally charged bodies are introduced, they will produce a divergence in the leaves. Now if the two bodies are removed, and another charged body introduced which produces the same divergence in the leaves as before, the charge on this body is twice as great as the charge on either of the first two.

Unit charge.

We say that a body has unit charge when it repels a body having an exactly similar charge with a force of one dyne, the bodies being one centimetre apart *in vacuo*.

The charge on any body can be measured by the force in dynes which it will exert on a body with unit charge at a distance of one centimetre.

The electrostatic unit charge, defined in this way, is quite different from the coulomb. A direct comparison of the two is not readily made but the coulomb is roughly equal to about 3×10^9 electrostatic units.

The *laws of force* between charged bodies are as follow :

- (1) Similarly charged bodies *repel* each other. Dissimilarly charged bodies *attract* each other.
- (2) The force between two charged bodies *in vacuo* is measured by the product of their charges divided by the square of the distance between them.

The second law, the law of inverse squares, was tested by Coulomb on his Torsion Balance. A more rigid proof of it,

however, can be deduced from the fact that no charge will remain in the inside of a conductor.

214. Proof of the law of force. Take any point O inside a charged conducting sphere. Through O draw any chord AOB . Now let a line POQ pass through O and revolve round the chord, so as to cut out a small area round each of the points A and B . Call these areas a and b .

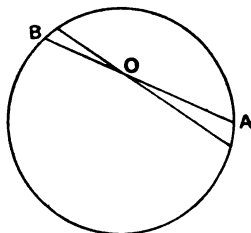


Fig. 173.

If we *assume* the law of inverse squares, then the force on unit charge at O due to the charge on $a = \frac{a\sigma \times 1}{OA^2}$ and that due to the charge on $b = \frac{b\sigma \times 1}{OB^2}$, σ being the surface density of the charge. But, since the areas a and b are proportional to the squares of their distances from O ,

$$\frac{a}{OA^2} = \frac{b}{OB^2}.$$

The forces due to the charges on the two parts a and b of the surface will therefore exactly counterbalance.

The whole surface of the sphere can be cut up into opposing parts in this way. The charge on the sphere will thus exert no force on a charged body placed inside it.

Now suppose the law of inverse squares *does not hold*, and that the force falls off faster with the distance than it would under the law. The parts far from O would have their influence decreased more than the parts near O .

Draw through O a plane perpendicular to the diameter through O , cutting the sphere into two parts, the cap and the base. Let the sphere be positively charged and a positive charge be placed at O . Then as shewn above, the push of the cap on the charge at O is exactly equal to the push of the base if the law of inverse squares is true; so that, *on our supposition*, the push of the cap would be greater than the push of the base, and the charge at O would be driven to the centre and would not go to the outside of the sphere if connected with it. This is contrary to the Cavendish experiment.

If on the other hand we assume that the force falls off *more slowly* than it would under the law of inverse squares, the base would get the advantage over the cap, with the result that a negative charge would not go to the outside of the sphere. This again contradicts experiment.

The law of inverse squares must therefore hold.

EXAMPLES

1. A charged ball is near a large earthed metal plate. Draw the lines of force and equipotential surfaces.

2. Draw the lines of force and the equipotential surfaces due to charges $+2$ and -1 on spheres of radius one centimetre placed 10 cm. apart.

3. Draw the lines of force between two spheres, one surrounding the other but not concentric with it.

4. Draw the lines of force in the following cases :

- (a) two positive equally charged spheres,
- (b) two equally charged spheres, one positive and one negative,
- (c) a parallel plate condenser.

5. A small positively charged sphere is placed near one end of a small insulated uncharged cylinder, and both are placed near the centre of a large spherical metallic shell connected with the earth. Describe in general terms the distribution of the charges on the bodies and sketch the lines of force of the system.

6. What will be the nature of the equipotential surfaces (1) due to a charged sphere, (2) between the plates of a large parallel plate condenser ?

7. Describe the action of the electrophorus and draw diagrams to illustrate the changes in the field of electric force as (1) the plate is brought down on to the charged ebonite cake, (2) the plate is earthed, (3) the plate is removed by its insulating handle.

CHAPTER XXIII

MATHEMATICS OF ELECTROSTATICS

215. Gauss's Theorem. In current electricity (Art. 123) we defined the flux across an area, or the number of lines of force passing through it, as the product of the area and the normal intensity of the field; and we saw that it became necessary to attach 4π lines to each unit pole. In static electricity it is usual to vary the method. To each unit charge we imagine one line or Faraday tube to be attached: these tubes cannot subdivide and can only end on unit negative charges: as a consequence it follows from arguments exactly similar to those in Art. 123, that the number of Faraday tubes which cross an area is to be measured by the product of the normal intensity and the area divided by 4π . That these two definitions are consistent in the case of a sphere surrounding a charge at the centre is readily understood; that it is true always was shewn by Gauss and the result is known as Gauss's Theorem. The usual form of statement is that the total normal induction over any closed surface is equal to 4π times the total interior charge; the total normal induction being defined as $\int \mathbf{N} d\mathbf{S}$ taken over the whole surface where $\mathbf{N}$ is the outward normal intensity at an element $d\mathbf{S}$ of surface.

Some important consequences follow readily.

216. Force due to charge on a sphere. Suppose a charge Q is uniformly distributed over a spherical conducting surface. All its Q tubes must by symmetry be radial. Imagine a concentric spherical surface, radius r , to surround this:

every tube must cut this normally and the intensity must everywhere on the surface be the same; but

$$\text{intensity} \times \text{area} \div 4\pi = \text{number of tubes};$$

$$\text{hence} \quad \text{intensity} = 4\pi \times Q \div 4\pi r^2 = Q/r^2.$$

In other words the force due to a uniformly charged sphere at any outside point is the same as if the charge were concentrated at the centre. (See also Art. 220 for alternative proof.)

217. Infinite plane. Consider a plane of infinite extent uniformly charged with surface density σ . Consider the force at a point P . Take a point P' directly opposite to P . Round PP' as axis imagine a little cylinder to be described of cross section α . This must enclose a charge $\alpha\sigma$ and the $\alpha\sigma$ tubes of force attached must be normal to the surface, half passing normally each end of the cylinder.

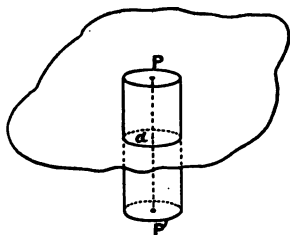


Fig. 174.

$$\text{Hence} \quad \text{intensity at } P \times \text{area} \div 4\pi = \frac{1}{2}\alpha\sigma,$$

$$\text{i.e. intensity} = 2\pi\sigma.$$

Hence the force due to a charged plane of infinite extent is $2\pi\sigma$ at all points, independent of distance. (See also Art. 221.)

218. Force outside a conductor. Suppose P is a point just outside the surface of a conductor and that the surface density near P is σ . Take any point P' in the material of the conductor just opposite to P and regard PP' as the axis of a short cylinder of cross section α .

Now (1) the Faraday tubes near P are normal to the surface, for lines of force must cut conductors at right angles, (2) no lines or tubes of force can exist in the interior of a conductor. Hence the $\alpha\sigma$ tubes attached to the area α on the surface enclosed by the cylinder

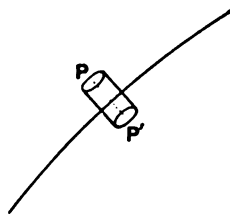


Fig. 175.

must all pass out normally through the end of the cylinder at P so that

$$\text{intensity at } P \times \text{area } a \div 4\pi = a\sigma,$$

$$\text{i.e. the intensity} = 4\pi\sigma.$$

This intensity is not due to the charge on the *conductor* only; it is due to that charge and to every other charge in existence. The force due to a plane of infinite extent is $2\pi\sigma$, but such a charge cannot exist without the corresponding opposite charge, the intensity due to which must also be $2\pi\sigma$. (See also Art. 222.)

219. The potential at a point due to a charge e at a distance d is $\frac{e}{d}$.

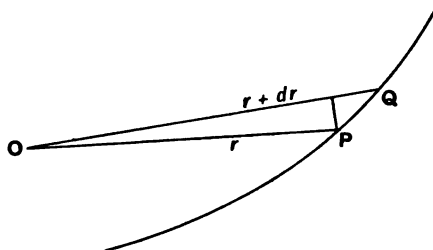


Fig. 176.

Let P be any point in the path along which a unit charge is brought up from infinity to a point, distance d from a charge e situated at O . Let Q be a point close to P on this path. Denote OP by r , OQ by $r + dr$; the work done by the electrical force in pushing a unit charge from P to Q = force $\times$ distance moved in direction of the force

$$= \frac{e \times 1}{r^2} \times dr.$$

Therefore total work in pushing a unit charge from A to an infinite distance

$$\begin{aligned} &= \int_a^\infty \frac{e}{r^2} dr \\ &= \left[-\frac{e}{r} \right]_a^\infty \\ &= \frac{e}{a}. \end{aligned}$$

Therefore work done against the electrical force in bringing a unit charge up to A from an infinite distance is $\frac{e}{a}$.

From this it follows that the potential at any point due to any number of different charges $e_1, e_2, e_3, \dots$ at distances $d_1, d_2, d_3, \dots$ is

$$\frac{e_1}{d_1} + \frac{e_2}{d_2} + \frac{e_3}{d_3} + \dots$$

220. Charged Sphere. At any external point the potential due to a uniformly charged sphere is the same as if all the charge were concentrated at the centre. Let σ be the surface density; we are going to find the potential at an external point O .

Let P be a point on the sphere, $OCP = \theta$, $OCQ = \theta + d\theta$.

If the figure revolves about the line OC , the element PQ will describe an annulus: the area of this = breadth $\times$ circumference

$$\begin{aligned} &= PQ \times 2\pi a \sin \theta, \quad \text{if } a = \text{radius of sphere,} \\ &= a d\theta \times 2\pi a \sin \theta. \end{aligned}$$

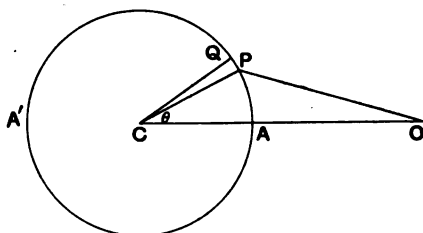


Fig. 177.

Therefore potential at O due to the charge on this ring

$$\begin{aligned} &= \frac{\text{charge}}{\text{distance}} \\ &= \frac{\sigma \times \text{area}}{OP} \\ &= 2\pi a^2 \sigma \sin \theta d\theta / r, \quad \text{if } OP = r. \end{aligned}$$

But

$$\begin{aligned} OP^2 &= OC^2 + CP^2 - 2OC \cdot CP \cos \theta, \\ r^2 &= l^2 + a^2 - 2al \cos \theta, \quad \text{if } OC = l. \end{aligned}$$

Differentiate this with respect to θ , and we get

$$\begin{aligned} 2r dr &= 2al \sin \theta d\theta, \\ \text{i.e. } \frac{\sin \theta d\theta}{r} &= \frac{dr}{al}. \end{aligned}$$

Hence potential due to ring

$$= 2\pi a^2 \sigma \frac{dr}{al},$$

$$\begin{aligned}
 \text{potential at } O \text{ due to sphere} &= \sigma \int_{OA}^{OA'} \frac{2\pi a}{l} dr \\
 &= \frac{2\pi\sigma a}{l} (OA' - OA) \\
 &= \frac{4\pi\sigma a^2}{l} \\
 &= \frac{\text{total charge}}{\text{distance from centre}}.
 \end{aligned}$$

Since at any point outside a uniformly charged sphere the potential is the same as if the charge were concentrated at the centre, it follows that the force also must be the same: for the magnitude of the force may always be found by differentiating the potential.

Hence the force outside a uniformly charged sphere
 $= \text{total charge} / (\text{distance from centre})^2$.

Now we know that there is no force inside a uniformly charged sphere: no work can then be done in moving a unit charge about inside such and the potential must be constant.

The potential then of a uniformly charged sphere is constant at all points and equal to the surface value, i.e. to $\frac{\text{charge}}{\text{radius}}$.

The capacity of a sphere is its charge divided by its potential: from this it follows that the capacity of a sphere is measured by its radius.

221. Infinite Plane. The force at any point due to a charged plane of infinite extent is $2\pi\sigma$.

Let O be the point: ON the perpendicular from O on the infinite plane.

Let OP be a radius vector from O to the plane making an angle θ with ON . Let Q be a point in NP near P so that $POQ = d\theta$.

Let the figure revolve round ON as axis: the line NPQ will sweep out the infinite plane, PQ a ring of area $2\pi NP \cdot PQ$.

Each element of this ring exerts a force at O : the resultant must be in the direction NO and its magnitude the sum of the resolved parts of the forces due to each element.

Hence force due to ring $= \sigma \cdot 2\pi \cdot NP \cdot PQ \cos \theta / OP^2$,

$$NP = h \tan \theta,$$

$$PQ = d(NP) = h \sec^2 \theta d\theta,$$

$$OP = h \sec \theta,$$

and

hence the force due to ring

$$= 2\pi\sigma \cdot d \sin \theta d\theta,$$

and the force due to plane

$$= \int_0^{\frac{\pi}{2}} 2\pi\sigma \sin \theta d\theta = 2\pi\sigma.$$

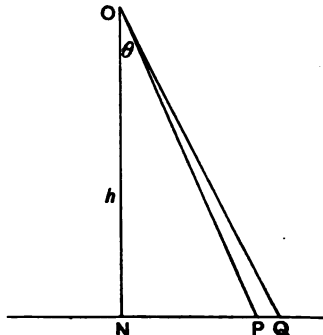


Fig. 178.

222. The intensity of the field just outside any conductor is $4\pi\sigma$. Suppose O is any point just outside a conductor. The surface density near O is σ . Consider a portion of the surface near O : compared with the rest of the conductor, this portion is to be small, but compared with the distance of O from it, it is to be great. This is evidently possible, for there is no limit to the closeness of O to the surface. Seen from O then this portion will appear infinite in extent. By Art. 217 we know that it will give rise to an intensity of $2\pi\sigma$ at O .

Now the resultant field at O is due to

(1) the force due to this area: call it F
($=2\pi\sigma$),

(2) the force due to other charges: call it F' .

Now consider a point O' , opposite to O and just inside the surface.

The resultant field at O' is zero, for there can be no force in the interior of a conductor: but this zero resultant is due to

(1) the force due to the area: this must be $2\pi\sigma$ inwards ($= -F$),

(2) the force due to the other charges, and this must be equal to F' , for O, O' are quite close together.

Hence F and F' must be equal and opposite.

The resultant field at O must therefore be of magnitude $2F$, or $4\pi\sigma$. It is normal to the surface.

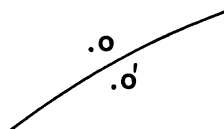


Fig. 179.

223. The capacity of a plane condenser. We shall suppose that the plates are separated by air and are so close together that seen from a point between them they may be regarded as infinite.

If the density of the charge on one surface is σ , that on the other is $-\sigma$. By Art. 221 or 217 the force at any point between due to either plate is $2\pi\sigma$: the resultant is $4\pi\sigma$. Hence the work done in carrying a unit charge across the gap is $4\pi\sigma t$, where t is the distance between the plates; this is the potential difference between them.

The capacity of an area A

$$= \frac{\text{charge on area}}{\text{potential difference}} = \frac{A\sigma}{4\pi\sigma t} = \frac{A}{4\pi t}.$$

224. Mechanical force on the plates of a condenser.

From Art. 217 it follows that the force on unit charge on one plate due to the charge on the other is $2\pi\sigma$; but the total charge on either plate is σA : therefore the total force on one plate due to the other is $2\pi\sigma \cdot \sigma A$.

Denoting this by $\mathbf{F}$, and the potential difference by $\mathbf{V}$, we have

$$\mathbf{F} = 2\pi\sigma^2\mathbf{A},$$

$$\mathbf{V} = 4\pi\sigma t,$$

whence

$$\mathbf{F} = \mathbf{A}\mathbf{V}^2/8\pi t^2.$$

225. Energy. To find the work necessary to charge a system of bodies.

Take the case of any conductor isolated from other bodies. Imagine it receives its charge gradually by a succession of small instalments. To bring up the first instalment from infinity needs no work for there is no charge to repel it: but to bring up any other, work will have to be done against the repulsion of the charge already supplied.

Let the final charge be $\mathbf{Q}$, the final potential $\mathbf{V}$. Then if at any time during the process the charge is $k\mathbf{Q}$ the potential must then be $k\mathbf{V}$; k being a fraction, less than unity.

Let the charge be increased from $k\mathbf{Q}$ to $(k + dk)\mathbf{Q}$ by the addition of the charge $dk \cdot \mathbf{Q}$. The work necessary to bring unit charge from infinity to the body is the potential, i.e. $k\mathbf{V}$; the work necessary to bring up $dk \cdot \mathbf{Q}$ is therefore $k\mathbf{V} \cdot dk \cdot \mathbf{Q}$. Hence the total work in increasing the charge from 0 to $\mathbf{Q}$ must be $\int_0^1 k dk \mathbf{Q}\mathbf{V}$, i.e. $\frac{1}{2}\mathbf{Q}\mathbf{V}$.

If there are several conductors in the field the same argument holds, if we assume that the charges are all received gradually at the same rate. Hence the energy of a set of conductors at potentials $\mathbf{V}_1, \mathbf{V}_2, \dots$ with charges $\mathbf{Q}_1, \mathbf{Q}_2, \dots$ is $\frac{1}{2}\mathbf{Q}_1\mathbf{V}_1 + \frac{1}{2}\mathbf{Q}_2\mathbf{V}_2 + \dots$.

The energy of a sphere of charge $\mathbf{Q}$ and radius a is $\frac{1}{2}\mathbf{Q}^2/a$.

226. Electrometer. The experiments described in Current Electricity and Magnetism have been largely quantitative; but no exact measurements have been taken in the section on Static Electricity. We have talked about unit charges and unit differences in potential and have not troubled to find out whether the measure of these things in the case of a piece of rubbed sealing wax is in thousandths or in millions. The matter is not of much importance to us: still we shall describe one method of obtaining fairly exact numerical results.

Fig. 180 is a diagrammatic sketch of an attracted disc electrometer. It consists of a condenser made up of two horizontal metal plates: only the edges of these are seen at AB and CD . The lower plate is fixed. The upper plate is cut into two parts:—an outside “guard ring” and an inside disc E , though the two are joined up together by a flexible wire w .

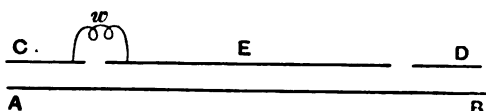


Fig. 180.

The disc E is supported from one beam of a balance so that the attraction between it and the plate AB can be measured when the condenser is charged.

Let us suppose that σ is the surface density and t the distance between the plates; then (Art. 223) the electric intensity between the plates is $4\pi\sigma$ and the potential difference (V) is $4\pi\sigma t$.

Also the attraction between one plate and unit charge on the other is $2\pi\sigma$ (cf. Art. 224): if then A is the area of the upper plate, the attraction (F) between it and the lower plate will be $2\pi\sigma \times A$.

From the equations

$$F = 2\pi\sigma^2 A,$$

$$V = 4\pi\sigma t,$$

eliminate σ and we get

$$V = \sqrt{\frac{8\pi F}{A}} t.$$

Hence the potential difference may be found in electrostatic units by measurements of force, distance and area.

A word must be added on the function of the guard ring. The formulae above were calculated on the assumption that the planes were infinite and that the field between the plates was uniform. Actually plates have edges and the lines of force near the edges have been shewn in fig. 171. But in any case the centre of the field between two plates is uniform and by use of the guard ring we cut off the edges and so avoid their disturbing effects.

227. Spheres. The following results for charged conducting spheres must be remembered.

(1) At any point outside a charged sphere, the force exerted on another charge is the same as it would be if the total charge on the sphere were concentrated at the centre. Hence,

(2) At any point on the surface of, or outside a sphere, the potential is the same as if the charge were concentrated at the centre.

(3) There is no force at any point inside a charged sphere, due to the charge on the sphere. Hence,

(4) The potential at any point inside is constant, and equal to the value on the surface (2).

The potential of an insulated sphere is thus equal to the charge divided by the radius.

(5) The capacity of a sphere is measured by the radius.

228. Capacity of a spherical condenser. Let a, b be the radii of the spheres A, B . Let a charge Q be given to A : it will induce a charge $-Q$ on the inner surface of B . If B is earthed it can have no charge on the outside and its potential must be zero. The potential of A due to the charge Q on itself $= Q/a$, and the potential of A due to the charge $-Q$ on $B = -Q/b$.

Hence potential of A

$$= Q \left(\frac{1}{a} - \frac{1}{b} \right).$$

Divide the charge by this

and we get for the capacity the expression $\frac{ab}{b-a}$.

If we imagine both radii to become very large while the thickness remains small and equal to t , we find that the capacity per unit area of a plane surface is $\frac{1}{4\pi t}$. (See also Art. 223.)

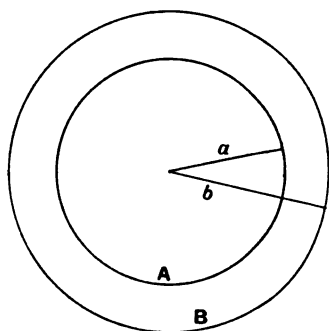


Fig. 181.

EXAMPLES

1. At what distance apart must two charges of 20 and 49 be placed to attract each other with a force of 45 dynes?
2. The repulsion between two equal charges placed 10 cm. apart is equal to the weight of 1 gram. What is the charge?
3. The force between two charges, one of which is double the other, is 30 dynes. If the distance between them is 8 cm., what is the amount of each charge?
4. Two equally charged spheres repel each other when their centres are half a metre apart with a force equal to the weight of 6 milligrams. What is the charge on each, in electrostatic units?
5. Three equal charges (16 units) are situated at the corners of an equilateral triangle (length of side=8 cm.). Find the resultant force on each, and the intensity at the centre.
6. Two charges of 4 and 9 units are placed 5 cm. apart. Find the intensity of the field at points in the line joining them 2 and 3 cm. away from the smaller charge.
7. An electrified metal ball is introduced into a dry glass tube closed at one end, and then, the tube being held in the hand, is brought near to the cap of the electroscope. What will the effect on the electroscope be if the exterior of the tube (1) is, (2) is not, covered with tinfoil?
8. Two small similar insulated balls, six inches apart, carry charges +3 and -7 respectively: they are brought in contact with one another, and then removed four inches apart; compare the forces between them before and after these changes.
9. Plot a curve to shew the decrease in force between two charges with the increase in distance.
10. Two unit charges are at *A* and *B*. What force will be exerted on a unit charge placed (1) midway between them, (2) at a point *C*, *ABC* being an equilateral triangle, if $AB=5$ cm.?
11. Three Leyden jars of capacities C_1 , C_2 , C_3 are arranged in cascade, i.e. with the inner coating of one connected to the outer of the next. Find the capacity of the arrangement when the inner coating of the first is charged and the outer coating of the last is put to earth. Would such an arrangement be of any practical use?
12. What is meant by charging Leyden jars in cascade? Three Leyden jars whose capacities are $\frac{1}{2}$, 1, $1\frac{1}{2}$ are arranged in cascade. What is the capacity of the combination?
13. A conducting sphere, of diameter 6, is electrified with 105 units: it is then enclosed concentrically within an insulated and unelectrified hollow conducting sphere formed of two hemispheres, of thickness $\frac{1}{4}$ and internal diameter 7. The outer sphere is then put to earth. Determine the potential of the inner sphere before and after the outer sphere is earth-connected.

14. Spheres A and B of radii 4 and 2 cm. have charges of 16 units each. What will be (1) their potentials, (2) their densities, (3) their capacities? If they are joined by a long thin wire, what will be (1) their total charges, (2) their potentials, (3) their densities?

15. Within a spherical vessel of brass 1 cm. thick, the external diameter of which is 14 cm., a brass ball 8 cm. in diameter is hung by a silk thread so that the centres of the two spheres coincide. If the ball is charged with 36 units of positive electricity, and if the potential of the vessel is 7, what is the potential of the ball?

16. If 100 ergs must be done in order to move a charge of 4 units from a place where the potential is -10 to another place where the potential is V , what is the value of V ?

17. Charges of 1, 2, 3 are placed at the corners of ABC , an equilateral triangle (side = 8 cm.); find the potential at the centre. If the charge at C were -3 , what would the potential at the centre be?

18. Two insulated and widely separated metallic spheres receive charges of positive electricity which raise their potentials to 4 and 5 respectively. The densities of the charges being in the ratio 4 : 9, compare the radii of the balls.

19. Find the work done in charging a sphere of 10 cm. radius with 50 units.

20. The capacity of a conductor is 20 c.g.s. units. What must be its charge in order that its energy may be 1000 ergs?

21. An insulated charged sphere, 3 in. in radius, is joined by a long wire to another 6 in. in radius; what is the relation between the energy of the charged sphere before and after connecting?

22. The potential of a sphere is 30 units, and its energy 80 ergs. What is (1) its charge, (2) its capacity, (3) its surface density?

23. How much energy is expended in carrying a charge of 50 units of electricity from a place where the potential is 20 to another where it is 30?

24. An insulated uncharged metal sphere is placed at a certain distance from another precisely similar but charged sphere. If the two spheres be connected by a fine wire, will (1) the quantity of electricity which passes, (2) the energy of the discharge, be affected by the distance between the spheres at the moment when the discharge takes place?

25. If the area of the surface of the attracted disc in a guard ring electrometer be 88 sq. cm., the attraction between the discs 2016 dynes, when the distance between them is 3 cm., find the difference of potential.

26. How would you use an electrometer to compare the E.M.F.'s of two cells?

27. An insulated metal plate, 10 cms. in diameter, is charged with electricity and supported horizontally at a distance of 1 millimetre below a similar plate suspended from a balance and connected to earth. If the attraction is balanced by the weight of one decigram, find the charge on the plate.

28. What loss of electrical energy occurs when a charged sphere shares its charge with another of the same radius? What becomes of this energy? Under what circumstances is the loss a maximum?

29. Find the electric intensity at a point due to a long cylinder uniformly charged.

30. Find the capacity per unit length of a cylindrical condenser.

31. Two spheres—radii r and $2r$ —widely separated are charged up to have surface densities of σ and $-\sigma$. Find the loss in energy when the two are joined up by a wire.

32. What is the capacity of a sphere radius 10 cm. which surrounds a concentric earthed sphere of radius 5 cm.?

33. Shew that the sectional area of a Faraday tube is inversely proportional to the intensity.

34. A spherical condenser is discharged by alternate contact. Prove that the charges decrease in geometrical progression.

35. Two spheres each of radius a carrying a positive charge e are a great distance d apart. Find the potential of each and the energy of the system. If the distance between them is halved, what would be the gain in energy?

36. If the electric potential over a certain region of the earth's surface increases with height at the rate of 100 volts for each metre above the surface, what is the charge per square centimetre of the ground?

37. How would you test whether two charges were, or were not, exactly equal?

38. A ball radius a is surrounded by a concentric spherical shell, internal radius b , external radius c . The inner ball has a charge q but is at zero potential; find the charges on the surfaces of the shell and the potential.

39. Two equal spheres, A and B , carry charges Q and $-Q$. They are widely separated. A third equal uncharged sphere C makes contact first with A , then with B and then with A again. Find the charges left on A and B .

40. Two very small spheres, each of mass 0.1 gm., suspended by equal light insulating threads are charged with 3 and 4 electrostatic units of positive electricity and are found to hang 2 cm. apart. Find the length of the suspending threads.

41. Shew that a conductor which has a charge of surface density σ is subject to a mechanical force of $2\pi\sigma^2$ on each unit area of the surface.

CHAPTER XXIV

SPECIFIC INDUCTIVE CAPACITY

229. The dielectric. An electrified ebonite rod exerts a pull on a pith ball near to it. Is there anything to connect the ball to the rod? A horse can pull a cart but then there are traces to connect the two. Are there any traces to connect the rod and the ball?

Or take Newton's example of the earth and the apple. There is a mutual attraction. No one knows how the two are connected, but Newton held it inconceivable that there was no connection between them.

Let us do a few simple experiments:

(1) Hold an electrified rod over an electroscope: can you cut off or modify the action between them by separating them with (a) insulators such as dry paper, sheet ebonite, (b) conductors such as a sheet of brass.

(2) Take a Leyden jar the coatings of which can be removed. Charge it up and let it stand. Then remove the coatings and connect them together. You may get a small spark. Now put them back and discharge the jar in the usual way. You will get a much larger spark.

The experiment indicates that the seat of the electrical energy is not in the conductors but in the medium separating them.

(3) Turn back to the Epinus condenser. Connect one plate to earth and the other to an electroscope. Give the latter a charge. Now insert between the plates a slab of wax or sheet of dry glass, or ebonite, being careful that these are not charged. (Ebonite may be discharged if necessary by

passing it quickly through a flame.) The fall of the leaves indicates that the potential difference is decreased and therefore the capacity of the condenser increased.

In wireless telegraphy, the message is sent by means of the electrical disturbance caused by the sparking between two balls. Just before the spark passes the medium between and around the balls would appear to be in a strained state. The release of the strain by the passage of the spark produces a throb which travels outward through the medium and is detected by the receiver.

230. Faraday's experiment. The last experiment shews us that the potential difference (and therefore the capacity) of a condenser depends to a large extent on the nature of the dielectric between the plates.

Faraday took two spherical condensers which were made exactly alike, but the dielectric in one was air while that of the other was shellac. After connecting the two insides together and the two outsides to earth, he charged the former and then removed the connection. The potentials of the two insides would thus be the same, and also that of the outsides (zero). The difference of potential between the two coatings would therefore be the same in each condenser. He then removed the outside spheres (each made up of two hemispheres) and tested the charge of the inner sphere. He found that the charge on the one from the shellac condenser was about double that of the other. Since the condensers were similar, and there was initially the same potential difference between their coatings, it followed that the capacity of the shellac condenser was about double that of the air condenser.

When he used petroleum instead of shellac, he found the capacity three times that of the other. Various other numbers were obtained by using different substances. These numbers measure what is called the specific inductive capacity of the dielectric, that of air being taken as unity.

The s.i.c. of hydrogen is slightly less than that of air. In glass the s.i.c. is about 7, alcohol 25 and water 80.

With this new knowledge of the dielectric constant some of our results require a little revision.

Faraday's experiment shews that the force in a medium, the S.I.C. of which is k , is $\frac{1}{k}$ times the force there would be in vacuo or air.

The force between two charges e, e' at a distance d is given by the formula $F = \frac{ee'}{kd^2}$.

Potential at a point, $V = \frac{e}{kd}$.

The energy formula remains unaltered, Art. 225.

Capacity, $kA/4\pi t$, Art. 223.

We shall not consider the cases in which the medium is not homogeneous : but shall merely state that the capacity of a condenser, the dielectric in which is partly air and partly of material the S.I.C. of which is k , is the same as it would be were the material replaced by air of $\frac{1}{k}$ times the thickness.

The mechanical pull between two plates with fixed charges is unaltered by the presence of a slab of dielectric such as wax ; provided that the wax does not fill the whole space. If it does fill the whole, the pull is $\frac{1}{k}$ times the air value.

It is important to remember that inductive capacity—or, as it is sometimes called, Dielectric Constant—is not a mere number. We say that the specific gravity of lead is 11 : but if we speak of its density we must say 11 grm. per c.c. If we say that the S.I.C. of wax is 3, it is only because there is no name for the unit of S.I.C. Some writers make a distinction between Specific Inductive Capacity and Inductive Capacity : the first being a mere ratio and equal to the I.C. of the medium $\div$ I.C. of air (cf. Specific Gravity), the second being the physical quantity for whose unit no name has been assigned.

EXAMPLES

1. Find the capacity of a condenser made of glass and copper foil. The area of the foil on each side is 50 cm. $\times$ 40 cm.: the thickness of the glass is 1.5 mm. and its S.I.C. is 6.

2. Two spherical condensers have internal radii of 5 cm. and 8 cm.: the internal radii of the exteriors being 6 cm. and 10 cm. The exterior shells are joined to earth; the inner spheres are connected together and receive a total charge Q . How is Q divided if the dielectric is air?

If now oil (S.I.C.=3) be poured into the first, how will the charges be affected?

3. Two oppositely charged pith balls are suspended near to one another by fibres of silk. Draw the lines of force between them. How will these lines be affected if a thick slab of wax be introduced between them? Will the attraction between them be altered?

4. Of two insulated metallic plates, A and B , which are placed near and parallel to each other, A is connected with the cap of an electroscope, and B receives a charge of electricity. Explain the behaviour of the gold leaves of the electroscope when a slab of unelectrified sulphur is introduced into the space between the plates.

5. A spherical condenser has an external radius of 20 cm. and is charged up to a potential of 50 electrostatic units when the interior sphere is connected to earth. If the interior is insulated and the exterior shell is earthed what is the loss in energy and charge?

6. Two pith balls are suspended by insulating strings from two supports, and oppositely charged. Describe the position they will assume, and explain how it would be affected by interposing between the balls (1) a plate of sulphur, (2) a sheet of copper.

7. A small ball of shellac suspended by a long silk fibre is passed through the flame of a spirit lamp, and an electrified ball is then brought near to it. Will it be attracted, and, if so, why? What is the object of passing it through the flame?

8. Two Leyden jars, whose capacities are 1 and 2, receive charges 3 and 4 respectively. Compare their combined electric energies before and after their knobs have been in contact.

9. A condenser is composed of two square plates each of 10 cm. side; the plates are 1 mm. apart. It is found that 500 ergs are needed to charge it to a certain difference of potential. Find the difference of potential and the charge on either plate.

10. Two Leyden jars are exactly alike, except that in one the tinfoil coatings are separated by glass and in the other by ebonite. A charge of electricity is given to the glass jar, and the potential of its inner coating is measured. The charge is then shared between the two jars and the potential falls to 0.6 of its former value. If the specific inductive capacity of ebonite be 2, what is that of glass?

11. Briefly describe and explain the phenomena observed in the course of the following experiment:

(a) An insulated body charged with electricity is held in a fixed position near the knob of an insulated uncharged gold leaf electroscope.

(b) A thick plate of glass is introduced between the charged body and the knob.

(c) The knob is put for a moment in electrical communication with the earth.

(d) The charged body is removed.

(e) The glass plate is removed.

12. A Leyden jar is charged to a potential of 1,200 volts. It is then connected to a second jar having twice the area of coating; the glass in the second jar is twice as thick as that of the first. Find the potential of the two jars.

13. Two spheres both of diameter d are connected to the prime conductor of an electrical machine. One of them is surrounded by a thin concentric shell (of radius d) which is earth-connected: the medium between the two is of s.i.c. 3. Compare the charges on the spheres.

14. A sphere, radius 5 cm., is surrounded by a coating of wax 1 cm. thick (s.i.c.=2). This in turn is surrounded by another coating of ebonite of the same thickness (s.i.c.=3). The whole is enclosed by a thin brass shell, radius 7 cm. If the outer shell is earthed, what is the capacity of the condenser?

MISCELLANEOUS EXAMPLES

1. If two exactly similar magnets were arranged with the intention of forming an astatic pair but with their axes not set exactly parallel, how would the combination behave when freely suspended?

2. Two long thin knitting needles are each suspended by two threads 30 cm. long attached at either end, so that the needles hang in a horizontal position and just touch along their whole length. The needles are then equally magnetised, after which they repel each other, so that the distance between them is 2 cm. If each needle weighs 5 grams and g is 980 cm. sec.⁻², calculate the strength of one of the poles of either needle.

3. Prove that the magnetic force exerted by a short magnet at a point A on the line passing through its centre and perpendicular to its axis is the same as the force exerted at a point on the axis, the distance of which from the centre of the magnet is $\sqrt{2}$ times the distance of A from the centre.

4. Two small magnets, each of length l and moment M , are placed with their axes in line and their centres at a distance r apart. Find the force of attraction or repulsion between them.

5. A bar magnet is suspended by a wire so as to hang horizontally. By how much must the top end of the wire be twisted for the magnet to be deflected 90° from the magnetic meridian, when for a deflexion of 30° it has to be twisted through 120° ?

6. A bar magnet is supported horizontally by a fine vertical wire. The magnet lies in the magnetic meridian when there is no torsion in the wire, and the top of the wire must be turned through 100° in order to deflect the magnet 15° from the meridian. The magnet is removed, remagnetised and replaced, and now the upper end of the wire has to be twisted through 150° to produce the same deflexion of the bar. Compare the moments of the bar in the two cases.

7. A short horizontal bar magnet is moveable so as always at its middle point to touch a horizontal circle, at the centre of which is a compass needle. Determine the positions of the magnet in which the needle's deflexion is the largest.

8. What is the function of the "sole" in the electrophorus?

9. Shew how the magnetic moments of two unequally strong magnets may be compared by mounting them in the manner of astatic needles (1) with like, (2) with unlike, poles together, and observing their oscillations when so mounted.

10. A short ebonite rod, with a small electrified knob at one end, is mounted so as to turn freely about its centre in a horizontal plane. In a horizontal line with this centre, and at distances from it of a quarter and half a metre respectively, are placed insulated balls that are also charged. The rod makes ten vibrations in a given time, but makes thirty vibrations in the same time if the balls are interchanged. Compare the charges on the two balls.

11. The coil of a tangent galvanometer is placed at right angles to the magnetic meridian and a steady current passes through it. The needle when set in vibration makes 5 oscillations in a given time, but only 3 in the same time when the direction of the current is reversed. Compare the magnetic force at the centre of the coil due to the current with that due to the earth.

12. In a B.A. bridge the wire has a resistance of 2.30 ohms, $K=5$ ohms, $X=3$ ohms. The wire is a metre long. Find the position of the jockey. If the cell has an E.M.F. of 1.2 volts and its resistance is negligible, find the possible error in the position of the jockey if the galvanometer has a resistance of 10,000 ohms and will detect a current of one microampere. By how much could this error affect the result obtained?

13. Shew that the potential due to a magnet at a distant point is $M \cos \theta / r^2$, where r is the distance from the centre, θ the angle the line joining the point to the centre makes with the axis and M is the moment.

14. When a circuit is completed the current does not immediately reach its final strength as given by the relation $C=ER$. Explain this and say how you would arrange an experiment to shew it.

15. Resistance coils are usually made by measuring off a length of wire, doubling it upon itself and then winding. What advantages are there in this method?

16. A wire of resistance r connects A and B , two points in a circuit, the resistance of the remainder of which is R . If without any other change being made A and B are also connected by $n-1$ other wires, the resistance of each of which is r , shew that the heat produced in the n wires together will be greater or less than that produced originally in the first wire according as r is greater or less than $R/\sqrt{n}$.

17. Power to the extent of 100,000 watts has to be carried to a distance of 5,000 metres with a loss not exceeding 5 per cent. Compare the cost of the copper mains if the current has a voltage of 100 with their cost if the voltage be raised to 2,000.

18. Describe as many methods as you can by which it would be possible to measure currents.

19. AB is a line of infinite length, O is a point on a straight line drawn at right angles to AB from a point C in AB . Unit charges are placed at $C_1, C_2, C_3, \dots, C_n$ in AB , where $CC_1, C_1C_2, C_2C_3, \dots$ subtend the same angle θ at O . Shew that the potential at O is

$$d^{-1} \cos \frac{1}{2}(\alpha + \theta) \sin \frac{\alpha}{2} \operatorname{cosec} \frac{\theta}{2},$$

where α is the angle subtended at O by CC_n and $OC=d$.

20. A condenser is built up of alternate layers of tinfoil and paraffined paper. If there are in all 101 sheets of foil, 12 cm. $\times$ 15 cm., and the thickness of the paper is .0025 cm., and s. i. c. = 3, find the capacity.

21. For use in spark photography a condenser made of glass covered on both sides with copper foil was made. The thickness of the glass was 3 mm.: the area of each plate 60 cm. $\times$ 60 cm. Sparking took place when the p. d. reached 25,000 volts: and lasted .000005 second. (s. i. c. = 5.)

Find (1) capacity of the condenser, (2) the horse-power at the instant of discharge.

22. A hand electric lamp is worked by three dry cells in series. It is said to have an E.M.F. of $4\frac{1}{2}$ volts on closed circuit and to give a light of 3 c. p. for 18 hours. Reckoning one watt per candle power, find (1) quantity of zinc consumed, (2) heat generated, (3) resistance of the lamp.

23. The resistance of a galvanometer is being found on a metre B. A. bridge. Its resistance is actually 2.35 ohms. The resistance of the wire is 0.51 ohm, the comparison coil is 3 ohms. The resistances of the key and battery are both negligible. The E.M.F. of the battery is 2.5 volts. The deflection of the galvanometer when the jockey is in its proper position is 23° . What is this position? Find what alteration there would be in the deflection if the jockey were moved 2 cm. nearer to the middle.

24. Prove that the charge on the body of highest potential in an electric field must be wholly positive.

25. Shew that the dimensions of quantity of electricity calculated electrostatically and electromagnetically are different. What assumptions lead to this result?

26. If the electric bells in a house were found to be out of working order, how would you try to find out what was wrong?

27. Two pith balls each weighing half a gramme are tied by silk fibres a metre long to the same support. They each carry the same charge and repel one another so that they remain 5 cm. apart. Find the charge.

28. A circular gold leaf, radius b , is laid on a charged conducting sphere of radius a . Prove that the loss of electrical energy due to the removal of the leaf is $\frac{1}{2}b^2E^2/a^3$, where E = charge on sphere and the capacity of the leaf is comparable with b , which is small compared with a .

29. A parallel plate condenser has a slab of dielectric (s. i. c. = $\mathbf{K}$) of thickness one-third of the distance between the plates, placed symmetrically between them. If $\mathbf{C}_1$ and $\mathbf{C}_2$ be the capacities of the condenser with and without the dielectric respectively, shew that $\mathbf{C}_1 : \mathbf{C}_2 :: 3\mathbf{K} : 2\mathbf{K} + 1$.

30. The tension of each surface in a soap film is t . If a is the radius of a soap bubble and p the atmospheric pressure, find the charge necessary to double the radius.

31. There is generally a considerable potential difference between two places in the air a few feet apart at different levels. Could we therefore get a current by joining up places at different levels by an insulated wire?

32. An earth-connected bucket of water has a small hole in the bottom through which water falls drop by drop. It is suspended in the air at a place where the potential is not zero. Will the bucket or the drops acquire any charge?

33. What justification have we for use of the term "quantity of electricity"?

34. In a polarised relay an iron armature governed by a spring is under the influence of a rod of iron fixed to a permanent magnet. Round this rod is wrapped a helix of wire through which the current received passes in such a direction as to tend to demagnetise the rod and so allow the spring to bring the armature into contact with a platinum stud. Draw a relay of this type and add an arrangement to alter the sensitiveness.

35. Make a careful diagram of the reversing key on an ordinary induction coil and explain the action.

36. Two spheres of radii r , r' are at a great distance d apart; shew that their capacities are increased in the ratio $d^2 : d^2 - rr'$.

37. The secondaries of induction coils are usually wound in sections carefully insulated from one another. Of what use is this?

38. Explain the fact that a spark is seen when a circuit, in which a current is flowing, is broken. How could this sparking be reduced?

39. In some forms of magnetos there is a fixed coil and a fixed permanent magnet: the flux through the coil is altered by the motion of iron pole pieces which connect the opposite ends of the iron core of the coil first with one pole of the magnet and then with the other. Draw an arrangement which would effect this and explain its action.

40. A battery consists of n cells in series; their E.M.F.'s are 1.1, 1.2, 1.3, 1.4, The internal resistance of each cell is 1 ohm. Find the value of n if a current of 1 ampere can be passed through an external resistance of 5 ohms.

41. A circuit is formed of a battery of constant E.M.F. with internal resistance 3.6 ohms and two wires of resistance 3.5 and 7.3 ohms respectively. Find the ratio of the amounts of heat produced in the wires when they are connected (1) in series, (2) in parallel.

42. An induction coil takes current of 2 amperes at 100 volts. If the discharge is at 500,000 volts and the efficiency is 30%, find the average current. If the actual duration of a spark is only .001 of the interval between successive sparks and there are 20 sparks per second, find the energy per spark and the rate at which energy is dissipated during a spark.

43. There is a series of $2n$ parallel plates. Each is of area $\mathbf{A}$ and is distant d from those next to it. By a revolving mechanical arrangement plates numbered 1, 3, 5, ... are connected to one pole of a battery of E.M.F. v , while plates 2, 4, 6, ... are connected to the other: they are then all disconnected, insulated and are finally joined up 2 to 3, 4 to 5, 6 to 7, ... by insulated connectors. What is the P.D. between plates 1 and $2n$? What charge would pass between them if they were connected? What is the energy and capacity of the arrangement just before discharge?

44. Four positive equal charges e are placed at the corners of a square. At the centre of the square is situated a negative charge. What must be the magnitude of this negative charge if it just holds the other charges in equilibrium?

45. Two equal charges e revolve at opposite ends of a diameter in a circle round a charge at the centre. If m is the mass associated with each charge, ω the angular velocity and a the radius, find the charge at the centre.

46. Devise a wiring arrangement to enable a lamp in the middle of a passage to be turned on or off by switches at either end.

47. A condenser is formed of two parallel plates. Prove that when the potentials of the plates are kept constant, the work done by the system in a small displacement is equal to the increase of the energy of the system.

48. The plates of a condenser are charged with a definite amount of electricity and then gradually separated. Find the mechanical work done. Find also the change in potential and the increase in electrical energy.

49. If the plates of a condenser are kept at constant potentials by joining them up to the poles of a battery, find (1) the mechanical work done in separating them by a definite distance, (2) the increase of electrical energy, (3) the energy supplied by the battery.

50. Shew that the result of the previous example illustrates the following general law. If the potentials of the conductors of a system are kept constant by batteries and an amount of mechanical work $\mathbf{W}$ is done in any displacement, then an amount of energy $2\mathbf{W}$ is drawn from the batteries and the electrical energy of the system is increased by $\mathbf{W}$.

51. A condenser of capacity c is connected up through a resistance r to a battery, the P.D. between the poles of which is v . Shew that after a time t the charge in the condenser is

$$cv \left(1 - e^{-\frac{t}{cr}}\right).$$

52. A microfarad condenser is discharged through a non-inductive resistance of 1,000,000 ohms. How long will it take for half the charge to pass?

53. A , B and $n-2$ other points are joined up in as many ways as possible by conductors each of the same resistance. A and B are also joined to the terminals of a cell. Prove that of these $\frac{1}{2}n(n-1)$ conductors only $2n-3$ carry currents, and that the current in one of these is double that in any of the rest.

54. A small conductor hangs by a string midway between two equal positive charges which are on the same level. Shew that the arrangement may become unstable when the conductor is charged either with positive or negative electricity, but that the minimum charge necessary to produce instability in the former case is twice as great as it is in the latter.

55. An electrified particle of mass m and charge e is shot off with velocity v in and at right angles to a magnetic field of strength H . Prove that it will describe a circle of radius mv/eH .

56. An electrified particle, mass m , charge e , is projected horizontally with velocity v between the horizontal plates of a condenser of charge σ per unit area. Prove that it will begin to describe a parabola and find the latus rectum.

PRACTICAL EXERCISES

1. Find the least E.M.F. that will drive a current through a dilute acid voltmeter with carbon electrodes.

2. Find the least E.M.F. that will drive a steady current backwards through a simple cell.

3. Find the changes in the current through a dilute acid electrolytic cell as the E.M.F. between the electrodes is gradually raised from 0 to 2 volts. The E.M.F. may conveniently be varied by use of a potentiometer bridge.

4. By reading ammeter, voltmeter and watch, plot a watt-hour curve for charging of an accumulator. Then discharge it at a reasonable rate and take corresponding readings. Compare the total intake of energy with the output.

5. A rod pointing North and South, placed an inch above the table, carries a current. Trace the lines of force on the table.

6. Thrust two magnetised needles through a cork (1) parallel with like poles together, (2) parallel with unlike together, (3) with axes at right angles. By the method of oscillation compare the moments of the combinations in the three cases. If M_1 , M_2 are the moments, separately verify that $\sqrt{M_1^2 + M_2^2}$ is the moment of the right-angled combination. From the first two arrangements find $M_1 : M_2$.

Find the directions of the axis of the combinations (3) and shew that the angle it makes with one of the needles is $\tan^{-1} \frac{M_1}{M_2}$.

7. Arrange the given metals in a list so that each is electro-positive to those following.

8. Place roughly in order of conductivity: iron, hot glass, warm glass, cold glass, ebonite, cotton, silk and flannel.

9. Find the magnetic axis of the given disc.
10. Compare the intensities of the fields at two places.
11. By means of a condensing electroscope determine the positive pole of a given battery.
12. Determine the magnetic condition of a given bar.
13. Determine the sign of the electrification of brass rubbed on paper.
14. Place the following materials in electric order, i.e. so that any when rubbed on one following it in the list is positively electrified. Fur, sealing wax, brass, silk, brown paper, glass, flannel, sheet rubber.
15. Verify experimentally that the conductivity of two wires joined in parallel is measured by the sum of their conductivities.
16. Compare the *E.M.F.*'s of two cells by arranging them (1) in parallel, (2) in opposition, using a tangent galvanometer.
17. Find a point on the axis produced of the given magnet at which the horizontal intensity is zero, and assuming $H = 18$ find the moment of the magnet.
18. The N. end of a long vertical magnet is placed on a horizontal table. Draw the lines of force, and calculate the strength of the pole assuming $H = 18$.
19. Compare the deflexion produced in a given galvanometer by the *E.M.F.* (1) when the whole current passes through the instrument, (2) when the galvanometer is shunted by a shunt of given resistance, and thence deduce the resistance of the galvanometer. The resistance of the current external to the galvanometer will be given.
20. Divide the given wire into two lengths so that when the divided lengths are placed in parallel their resistance as compared with the resistance of the undivided wire is 5 : 36.
21. Find the number of turns of wire on the given ring.
22. Find the resistance of a tangent galvanometer, a coil of known resistance being supplied and a cell of negligible resistance.
23. The given wire has a constant current flowing through it. Find two points on it such that their difference of potential shall be half that between the terminals of a given cell on open circuit.
24. Being provided with a Daniell cell, *E.M.F.* 1·08 volts, and a coil of known resistance, determine the reduction factor of the given tangent galvanometer.
25. Find the maximum strength of pole that can be induced in the given bar by hammering it when in a vertical position.
26. Employ the method of oscillation to determine the total horizontal magnetic force at each of the places *A* and *B*. Assume the value of H .
27. Determine the mean coefficient of increase of resistance of the given wire on being heated from 0° C. to 100° C.
28. Test the given wire for uniformity of resistance.

29. The E.M.F. of a copper iron junction with one temperature at 0°C . and the other at about 100°C . is too small to be measured by an ordinary millivoltmeter. It is, however, possible to obtain satisfactory results by the use of a reflecting galvanometer of high resistance. To do this, join the copper leads to the galvanometer. Keep one junction in a bath of boiling water and the other in a bath kept at 0°C . by ice and water. Start with the hot junction at the highest temperature you intend to employ. If you find the light spot comes off the scale, insert a resistance in the galvanometer circuit of such magnitude that the spot is near the end. Read the exact deflection and temperature. As the temperature falls other readings must be taken. Tabulate scale reading and temperature. Assume that the E.M.F. is proportional to the deflection. To get the actual electromotive force it is necessary to know the resistance of the circuit and the deflection produced for some known current.

30. Make up a Daniell cell and find the P.D. between its poles by a voltmeter:

- (1) immediately it is set up;
- (2) after standing a few minutes;
- (3) while ringing an electric bell;
- (4) after it has rung the bell for two minutes.

Take a Leclanché (or dry) cell and repeat (2), (3), (4) with it.

31. To trace the lines of flow of a current across a sheet of foil. Fix the foil to a drawing board with a sheet of paper underneath: use two drawing pins as terminals. Find first of all lines of equipotential. To do this drive a needle *A* anywhere through the foil and join it by a fine wire through a delicate galvanometer to a second needle *B*. Move *B* about till a point *C* is found on the foil, such that no current flows through the galvanometer on making contact. Prick through the foil at *C* into the paper. Find a series of points in the same way, all at the same potential as *A*. These may be joined up on the paper later. Proceed to find other similar lines.

32. Make some pole-testing paper and use it to find the positive pole of a cell made from acid, iron, and copper. Try to make and test cells with other pairs of metals.

33. Dip the ends of copper wires leading from the terminals of a battery into (a) solutions of copper sulphate, (b) dilute acid, for a few seconds. Note the appearance at each end and so give rules for finding positive and negative poles of a battery.

34. By placing a magnet in a water bath near a magnetometer, find how its moment is dependent on its temperature.

35. Find by the method of substitution the resistance of a coil. (Arrange a galvanometer, the coil and a cell in series. Note the current. Now replace the coil by a resistance box and arrange to get the same current.)

APPENDIX

ELECTRO-CHEMICAL EQUIVALENTS.

Aluminium	·000094	Hydrogen	·0000104	Silver	·001118
Copper	·000328	Oxygen	·000083	Zinc	·000338

E.M.F. OF CELLS in volts.

Daniell	1·08	Grove	1·9	Leclanché	1·4
Clark	1·43	Bunsen	1·9	Chromic acid	2·0

SPECIFIC RESISTANCE in microhms.

Aluminium	2·8	Iron	9·6	German Silver	120
Copper	1·6	Mercury	94	Silver	1·5

SPECIFIC INDUCTIVE CAPACITY (Air, 1).

Water	80	Petroleum	2	Ebonite	2—3	Paper	2·5(?)
Alcohol	25	Glass	6—10	Wax	2	Shellac	3

COPPER WIRE.

Gauge	14	18	22	26
Diam. in mm.	2·03	1·21	·71	·45
Yds. per lb.	17	48	140	340
Ohms per lb.	·08	·64	5·5	32

Gauges 14 and 18 can safely carry 10 and 4 amps. respectively.

Dimensions of Electrical Quantities.

The following table gives the dimensions of some of the principal magnetic and electrical units in terms of Mass, Length, Time, and a fourth unit. On the c.g.s. electrostatic system this fourth is specific inductive capacity, denoted by k : these dimensions may be worked out by reference to Articles 213, 32, 46, 199, in order. On the c.g.s. electromagnetic system the fourth unit adopted as a fundamental from which to derive others is permeability μ . Work them out in the following order, pole strength, Art. 4, current, Art. 138, charge, P.D., resistance, &c.

If we equate the dimensions of any quantity on the two systems, we find that $k^{-\frac{1}{2}}\mu^{-\frac{1}{2}} = LT^{-1}$. Now LT^{-1} indicates a velocity, V . A careful comparison of values on the two systems gives a value 3×10^{10} cm./sec. for V . To convert measures from one system to the other we must multiply by some power of V . Thus one electrostatic unit of resistance is equal to $3^2 \times 10^{20}$ electromagnetic units.

	Electrostatic				Electromagnetic				Ratio (i):(ii)	Practical unit	c. m. units
	k	M	L	T	μ	M	L	T			
Charge	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	V	coulomb =	10^{-1}
Current	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-2	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	V	ampere =	10^{-1}
P.D.	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-2	V^{-1}	volt =	10^8
Resistance	-1	0	-1	1	1	0	1	-1	V^{-2}	ohm =	10^9
Capacity	1	0	1	0	-1	0	-1	2	V^2	farad =	10^{-9}
Magnetic force	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-2	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	V	gauss =	1
Magnetic flux	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	-2	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-1	V	maxwell =	1
Pole strength	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-1	V^{-1}		
Inductance	-1	0	-1	2	1	0	1	0	V^{-2}	henry =	10^9

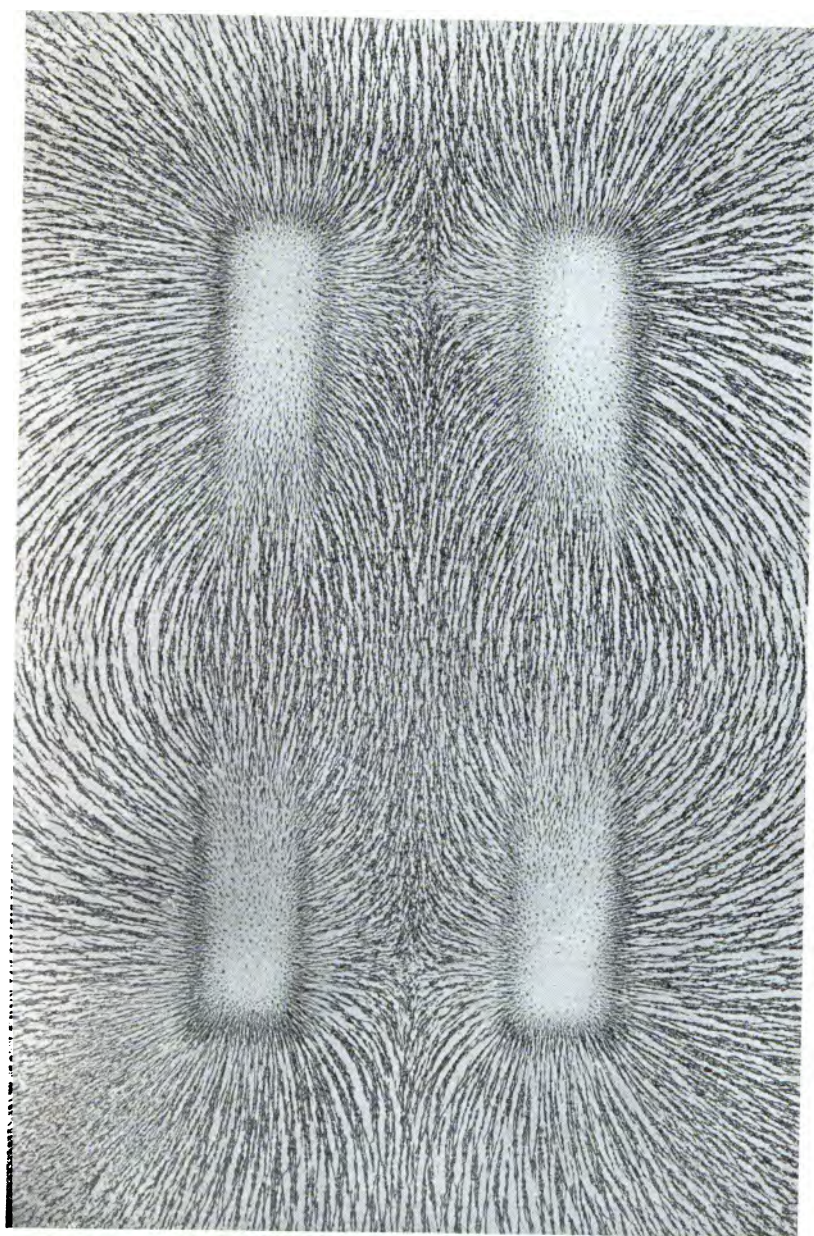


Fig. 128.

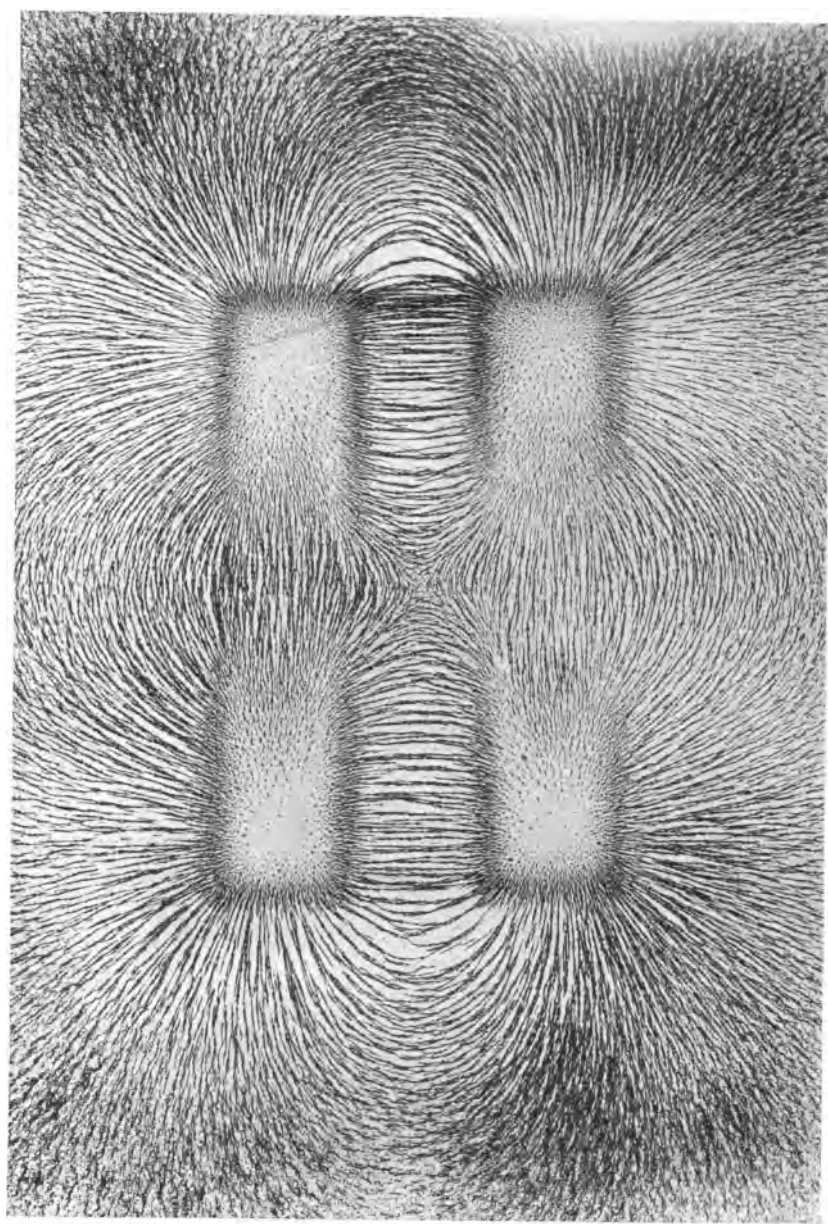


Fig. 129.

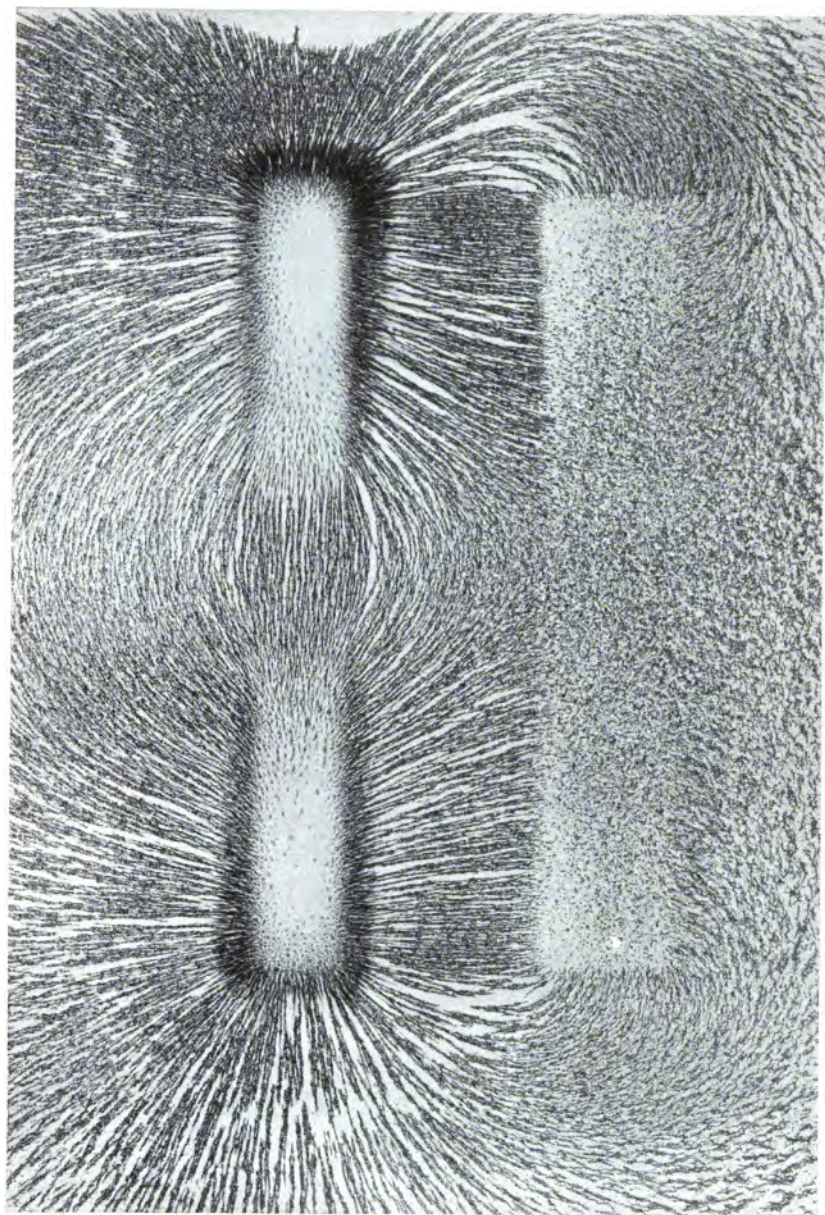


Fig. 130.



Fig. 131.

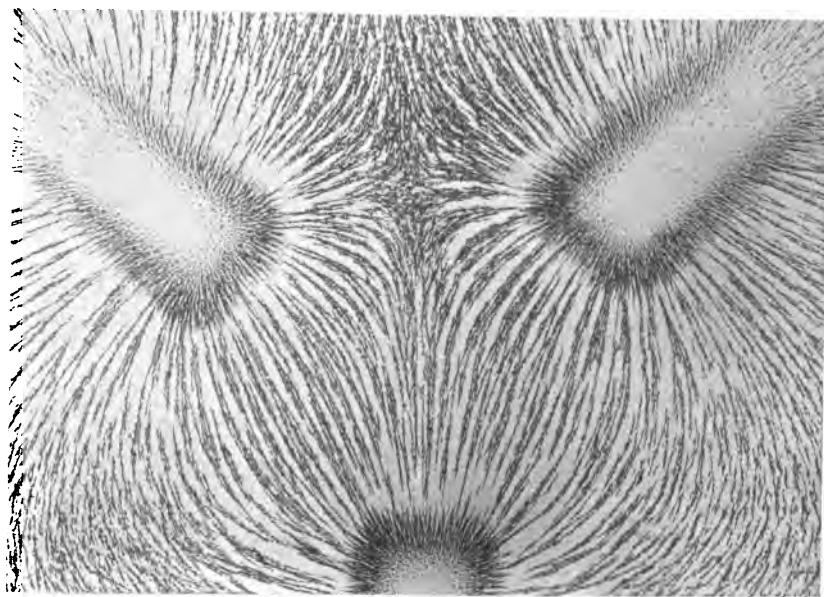
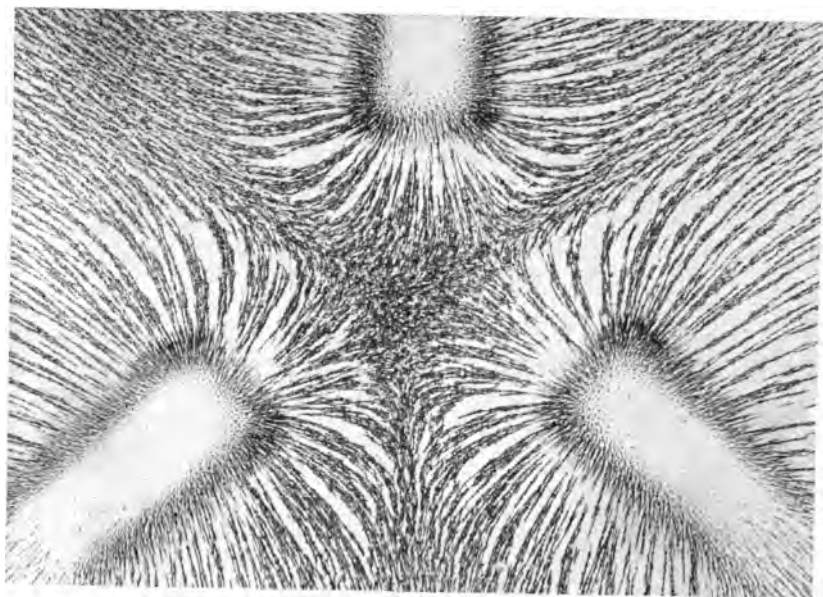


Fig. 132.

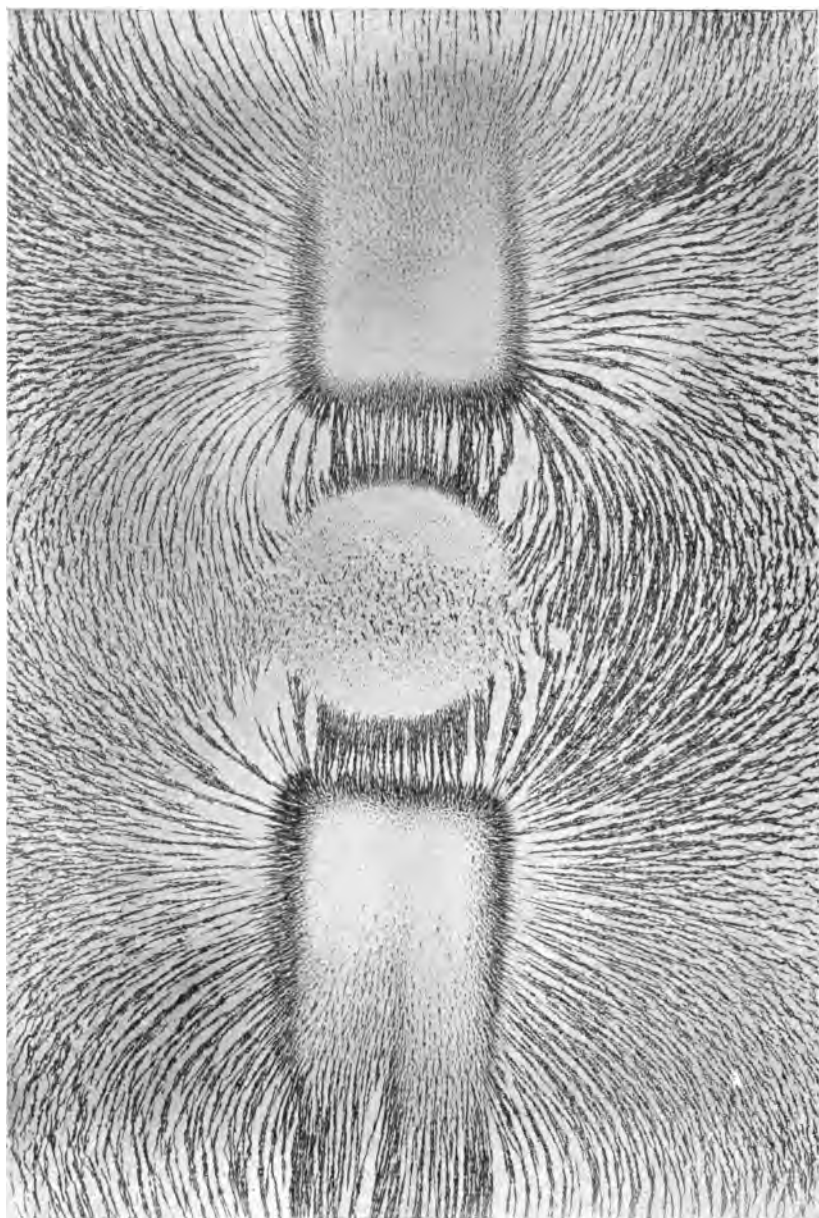


Fig. 133.

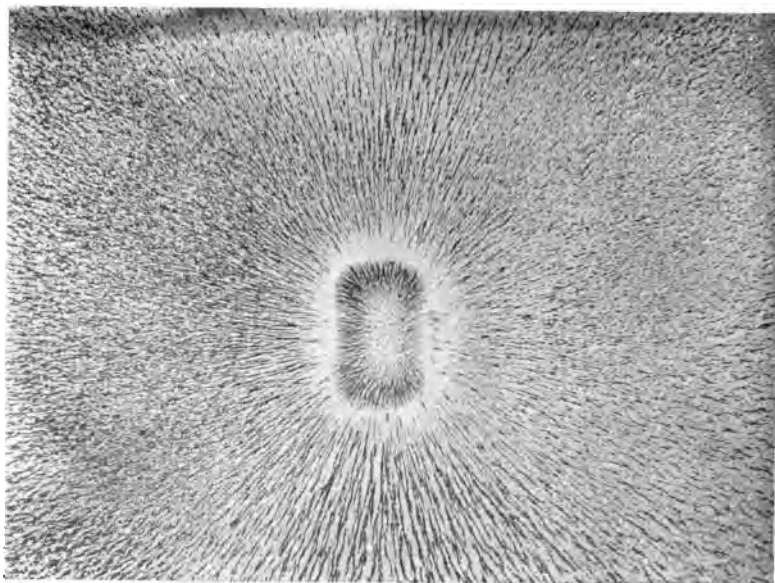


Fig. 134.

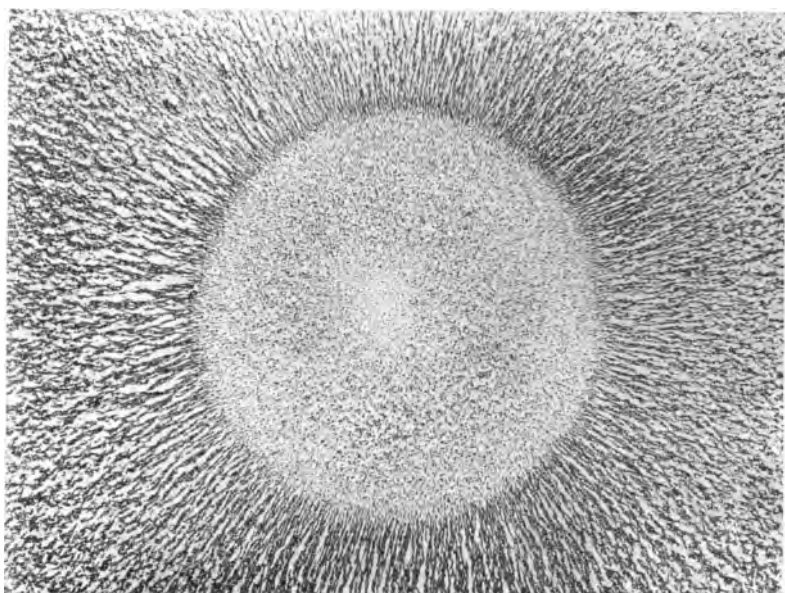


Fig. 135.

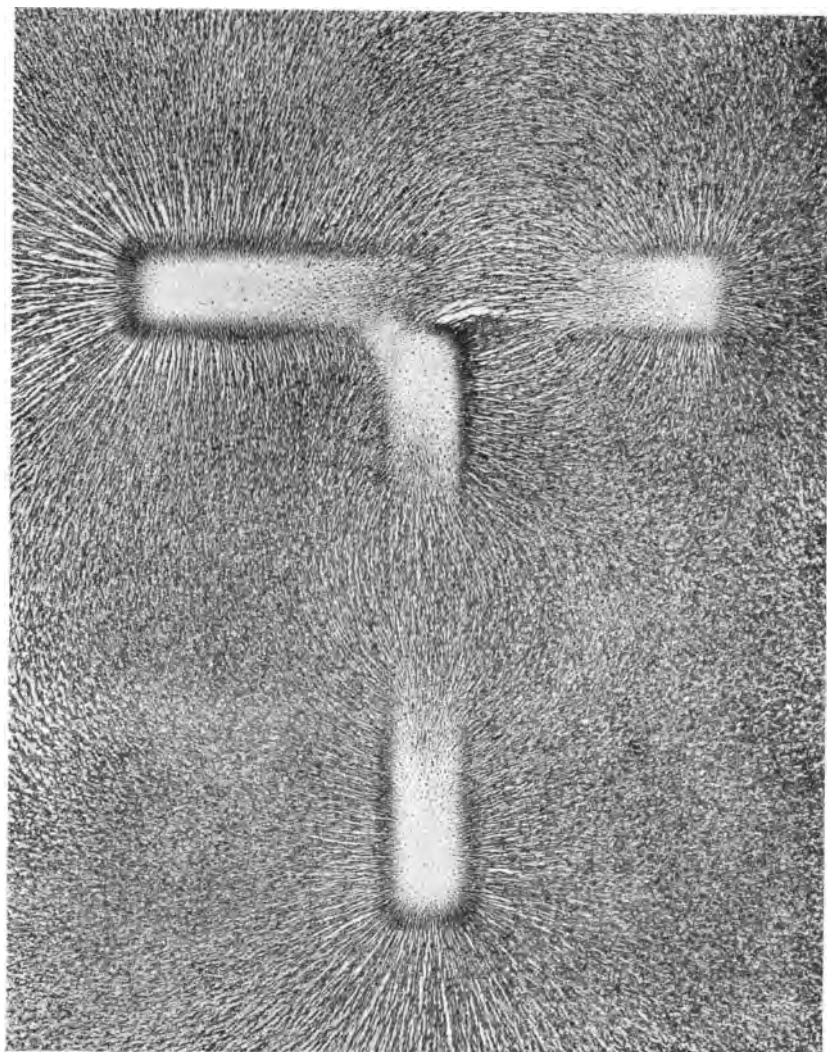


Fig. 136.

ANSWERS

PAGE

- 27.** 2. .047 grm. 3. 16760. 6. 1740 coulombs, 2.4 amp.
7. .42 amp., .85 amp.
- 28.** 9. 3.1 amp. 10. .00037. 11. 8.03; 31.8. 15. 29.8 mins.
16. 8.91 grm. per c.c.
- 40.** 1. 9.3. 2. 15; 714 cal. 3. 2.4 degrees. 4. 3×10^{-7} , 1×10^{-6} .
5. 1.1, 1.47. 6. 6s. 7d. 7. 6 h. 57 m. 8. 1.16.
9. 2571 coulombs. 10. .524, 102 kg. cal., 9.1 °/o. 11. 3.28.
- 41.** 12. 20 degrees. 13. 1:1.28. 14. .93°. 15. .359, 2.2.
17. 280, 2.4 %/o. 18. 52,000 joules. 19. .3°, 1.6 kw.
- 47.** 1. 8, 6, 3.2 amp. 2. $\frac{1}{2}$ ohm. 3. 440 ohms, 7.5 amp., 14.7 ohms.
4. $4\frac{1}{2}$, $1\frac{2}{3}$ ohm. 5. 6, 8 ohms. 6. 2.04 ohms. 7. 9.9, 23.5 ohms.
8. $1\frac{1}{3}$ ohm; 1, 1, 2 amp., 4, 3 volts. 9. $1\frac{2}{3}$ ohm. 10. 231, 286.
- 48.** 11. $r_1:r_2$. 13. 1250 ohms. 14. 59.5 volts, 4.9 ohms.
15. .017 volt. 16. $\frac{1}{2}$. 17. .00202 ohm. 18. .612 amp.
19. 153. 21. 16, 20. 22. 7.5; 5; 3.75 amp. 23. 1.316.
24. 5.12. 25. 5.75×10^8 , 12 m. 10 s.
- 49.** 26. .48. 27. $\frac{1}{2}$ or $1\frac{1}{2}$ amp.; 2.5 ohms, 1.25 volts. 28. 8571.
29. 1:1.01. 30. $\frac{1}{2}$ ohm. 32. $2\frac{2}{3}$ k. 34. .686.
- 55.** 1. 2.4 volts, .5 ohm. 2. 5 volts, 3.25 ohms. 3. 7.5 ohms.
4. .94 volt.
- 56.** 5. .232 ohms. 7. .353 amp. 8. 9 volts.
9. 920 ohms. 10. .458 amp. 11. .142 ohm.
12. 3.005 amp.; .64 volt; .32 ohm, 1.6 volt. 13. 1.2 volt.
15. $R = 2r$. 16. 1 volt. 17. 2.07 volts, .67 amp.; 4.27 volts, .427 amp.
18. .05, .55 volt.
- 65.** 2 17. 4. .4, .084 ohms, 9.03 watts. 5. 2.86 degrees.
- 66.** 13. .202 grm.; .197 grm. 16. .75, .039, .118 amp.
18. 1.66, 1.96 amp. 19. .43 amp. 20. $\frac{1}{11}$, $\frac{1}{31}$, $\frac{1}{17}$, $\frac{1}{13}$ amp.
21. 7.2 amp.

PAGE

- 67.** 22. '19, 31.5, 2.6 amp. 23. 6, 3 joules; 1, $\frac{1}{2}$ amp.
 25. $\frac{1}{4}$ volt. 26. One reversed. 27. 6 ohms.
- 81.** 4. 84.2, 50, 13.8; 2.2, 1.2, 2.5%. 6. $\frac{4}{3}$, $\frac{1}{3}$ amp. 8. $\frac{3}{4}$ volt.
- 82.** 9. '0013 volt. 10. 187.8, 227.6, '18 volt. 11. 1.15 volts.
 12. 1.5 volts. 13. -25, 1.45, '95, '25 volt. 15. '0024 volt.
- 93.** 1. '65 volt. 2. '40 amp. 3. 80°. 4. 17.4. 7. 35°.
- 94.** 8. '02 ohm. 12. '0375. 20. $\frac{1}{4}$ ohm.
- 95.** 24. '0066, '000936, '0042 amp.
- 102.** 1. 29 ohms. 2. '079 ohm. 4. '0032 volt. 7. 18.9 ohms.
- 122.** 18. $1VF$, 1.1×10^{-3} volts. 20. '000157 volt. 21. 3.02 maxwells.
 22. 1.9×10^{-5} amp., '037 erg, 6.6×10^{-5} dynes. 23. '603 amp.
- 133.** 1. 5.2×10^{-4} , 2.6×10^{-4} coulombs. 2. 860 max., 3.4×10^{-6} coul.
 3. 2700 max.
- 134.** 16. '377.
- 139.** 3. 13.9 amp. 4. 37.6 maxwells. 6. $4i(a^2 - d^2)^{\frac{1}{2}}/a^2$.
 9. $6\pi^2 r^4 i^2 / d^4$. 10. $\frac{2i}{d}$. 11. $2Mi/(r^2 - l^2)$.
- 172.** 1. 36. 2. 2.5 dynes. 3. 2.04, '51 grm. wt.
 4. 172 cm. outside, '32 dyne.
- 173.** 6. 35.7. 7. 100, 25. 8. 45.1 or 34.9. 9. 17. 10. '00080.
 11. 43.7. 12. '029, 1.05, '052. 13. '367. 14. 40 dyne cm.
 15. 1296.
- 174.** 16. 1:2.37. 17. '01 gauss. 18. 5°. 19. 40° 54'.
 20. 1:2.74. 21. 52.8. 22. $Mm\sqrt{5/d^3}$.
 24. 60,000; 1.77 sec., 1.12 sec. 25. 4 dyne cm. 26. '2, '505.
- 175.** 27. 1:1.33. 28. 206, '175. 29. 1351, 1780.
- 181.** 2. 171 ergs. 3. 127. 4. 5 gaussess, '35 maxwell.
- 227.** 1. $4\frac{2}{3}$ cm. 2. 5414. 3. 31. 4. 121. 5. 6.93, 0.
 6. 0, 1.8. 8. 7:3. 10. 0, '069. 11. $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$.
 12. $\frac{3}{11}$. 13. 33, 5.
- 228.** 14. 4, 8; $\frac{1}{4\pi}$, $\frac{1}{\pi}$; 4, 2. 21. $\frac{1}{3}$, $10\frac{2}{3}$; $3\frac{1}{3}$; $\frac{1}{3\pi}$, $\frac{2}{3\pi}$. 15. 10.
 16. 15. 17. 1.3, 0. 18. 9:5. 19. 125. 20. 200.
 21. 9:1. 22. $5\frac{1}{3}$, '178, 13.4. 23. 500. 25. 72. 27. 35.
- 229.** 31. $48\pi^2 r^3 \sigma^2$. 32. 20. 38. q ; $cq(a-b)/ab$; $q(a-b)/ab$.
 39. $Q/8$, $Q/4$. 40. 32.7.

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- 233.** 1. 6365 cm. 2. 3:4, 26Q passes from second to first.
 5. 25,000 ergs, 1000 units. 8. 51:49. 9. $2\sqrt{\pi}$, $500/\sqrt{\pi}$.
- 234.** 10. 3. 12. 600 volts. 13. 6:1. 14. 40·7.
- 235.** 2. 18. 5. 270° . 6. 1:1·6.
- 236.** 10. -1:7. 11. 8:17. 12. 37·5 cm., 83 cm., 108 ohm.
 17. 1:400.
- 237.** 20. $1·7 \times 10^6$ cm. 21. 4770 cm., 4400 H.P.
 22. 44 grm.; 46,000 cal.; $6\frac{3}{4}$ ohms. 23. 44 cm., $1^\circ 7'$. 27. 17·5.
 30. $4\sqrt{\pi a^3(12t+7pa)}$.
- 238.** 40. 10. 41. 78:1. 42. 00012 amp. : 3 joules : 60 kw.
- 239.** 43. nv ; $\mathbf{A}/4\pi d$; $n\mathbf{A}v^2/8\pi d$; $\mathbf{A}/4\pi nt$. 44. $e(1+2\sqrt{2})/4$.
 45. $ma^3\omega^2/e+e/4$. 52. 69 sec.
- 240.** 56. $mv^2/2\pi\sigma$.

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